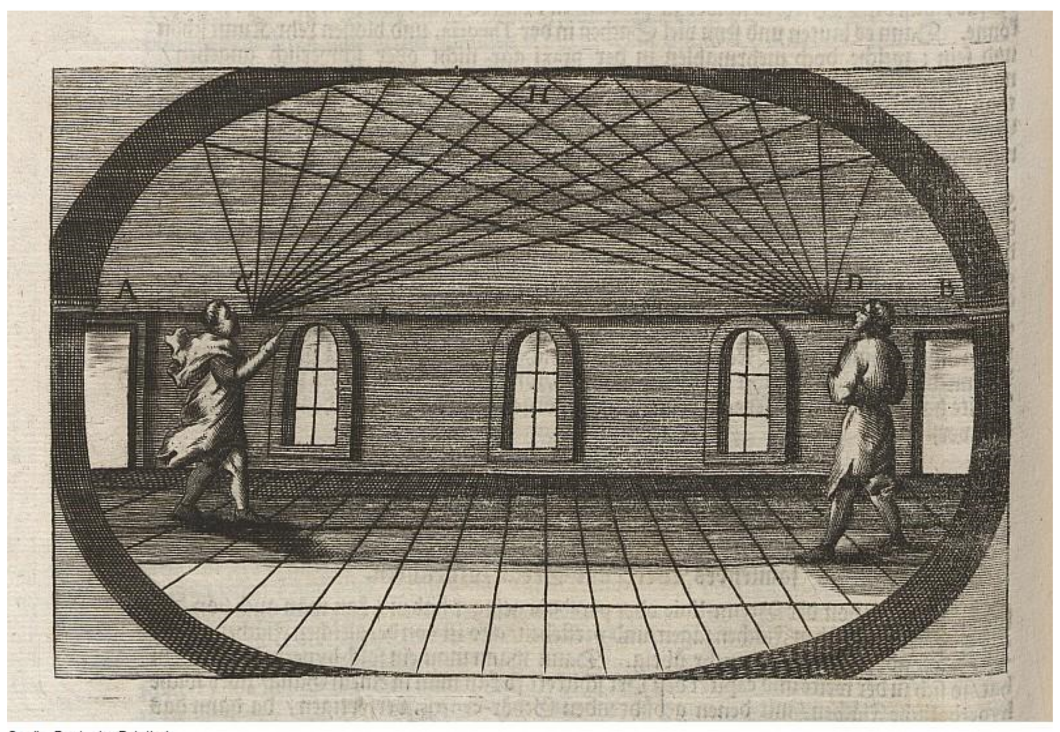
REPRESENTING

FROM ANTIQUITY TO OUR TIMES

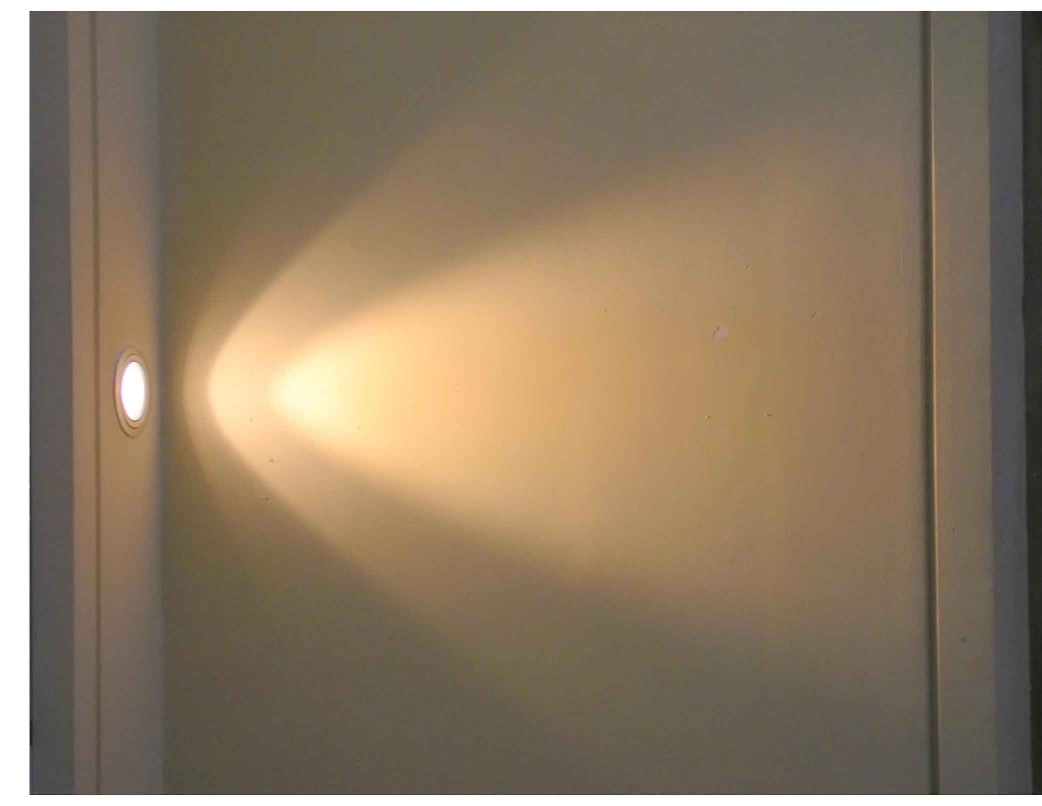
THE ADVENTURE OF CONICS



Quelle: Deutsche Fotothek

Echo chamber using a geometrical property of ellipse

Parabolic mirror of solar oven in Mont-Louis

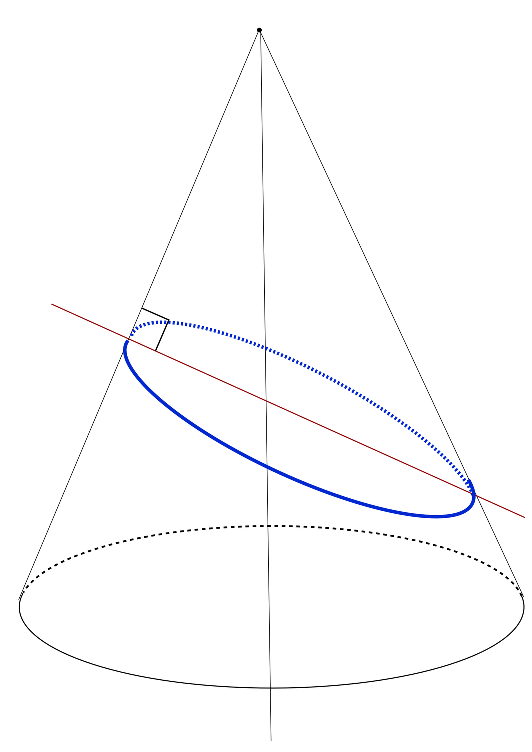


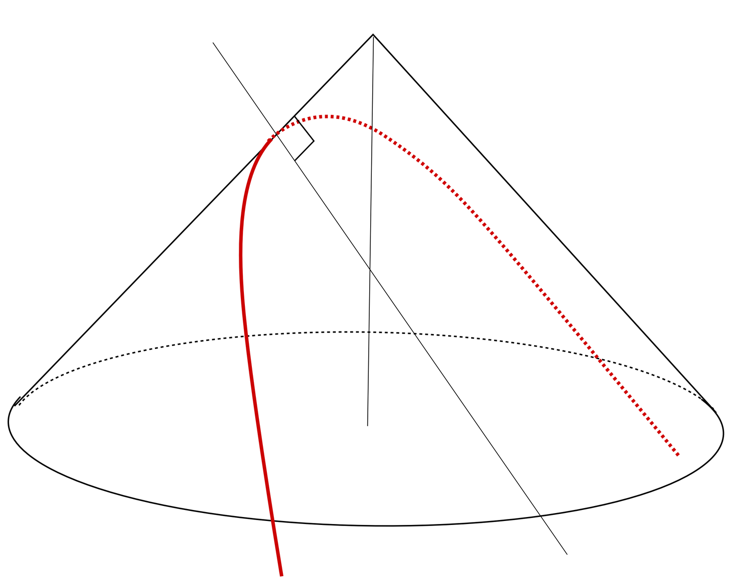
Hyperbolic trace of a light cone on a plane

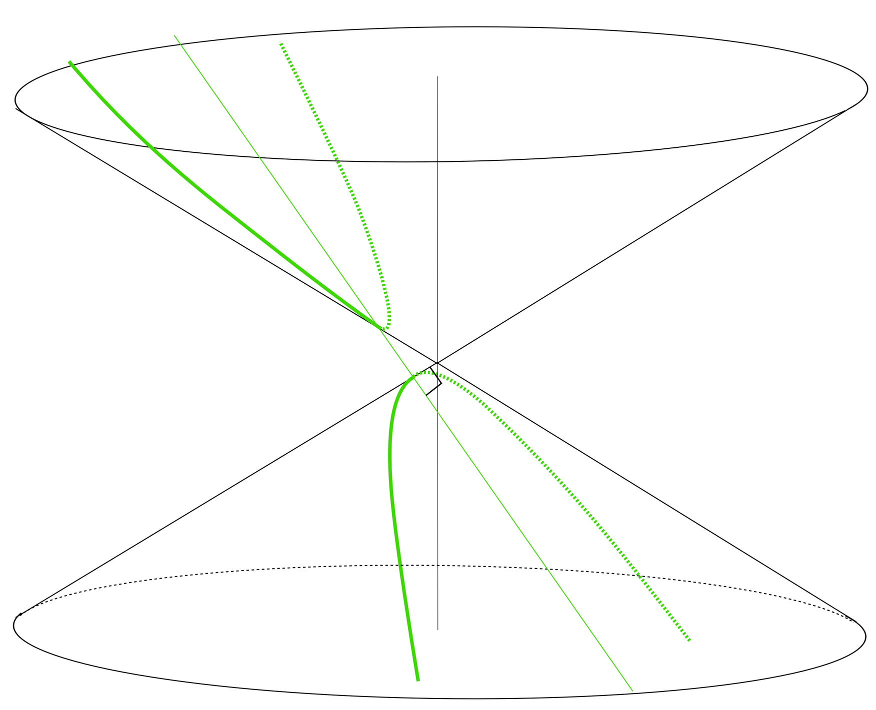
The first written evidence dealing with the intersections of cones and planes came in a document of **Menaechmus** (-380, -320). He proposed a solution to the problem of doubling a cube, also called the Delian problem. Being unable to obtain the solution by means of a ruler and compass, he proposed using the intersection of two conics to obtain the solution of a more general problem: that of inserting two geometric means between two given magnitudes.

Cone cut by a plane perpendicular to a generator

For **Menaechmus**, a conic is defined as the intersection of a cone of revolution and a plane perpendicular to one of its generators.
The **parabola** corresponds to the particular case of a cone with a right angle at the vertex (the plane of intersection is parallel to a generator).
Hyperbolas correspond to obtuse angled cones
Ellipses correspond to acute angled cones.
For Menaechmus, the equations that characterise conics are replaced by equalities of areas.

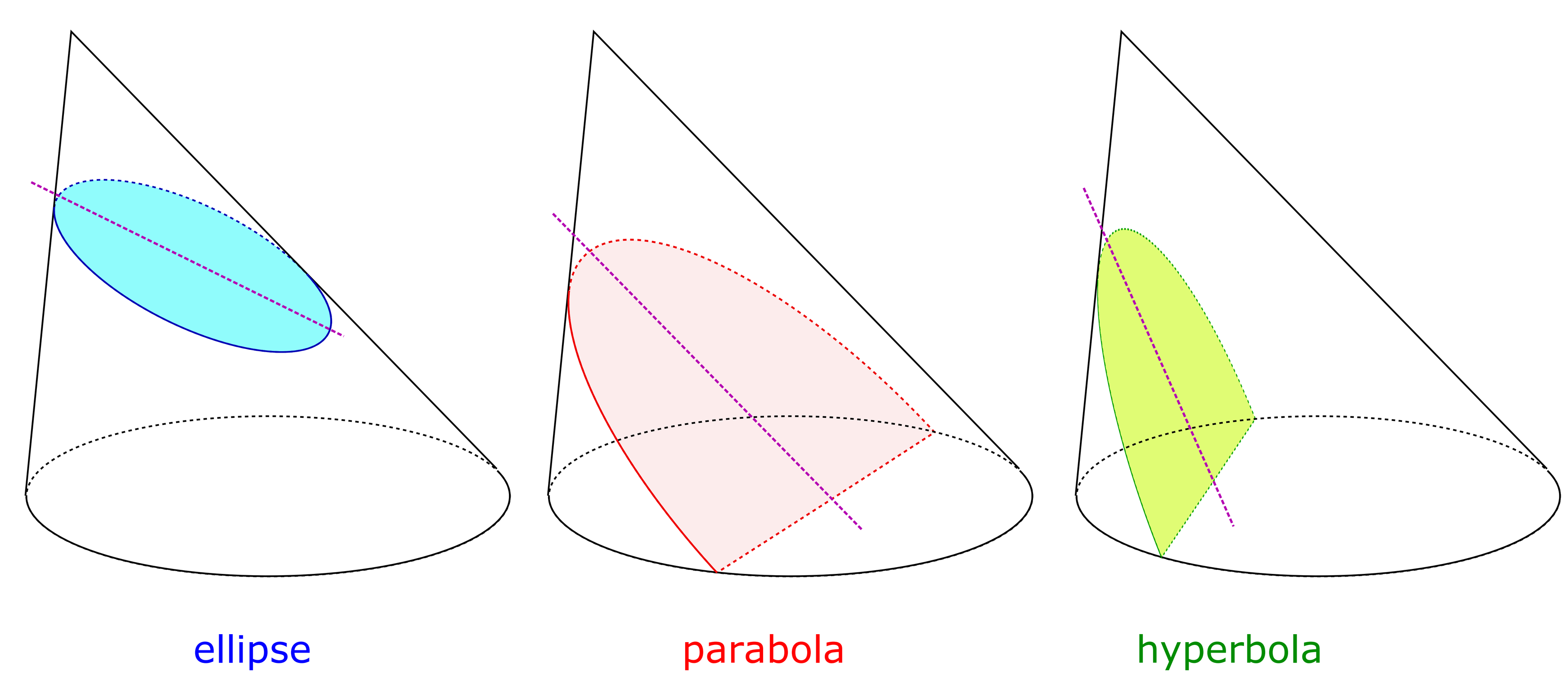




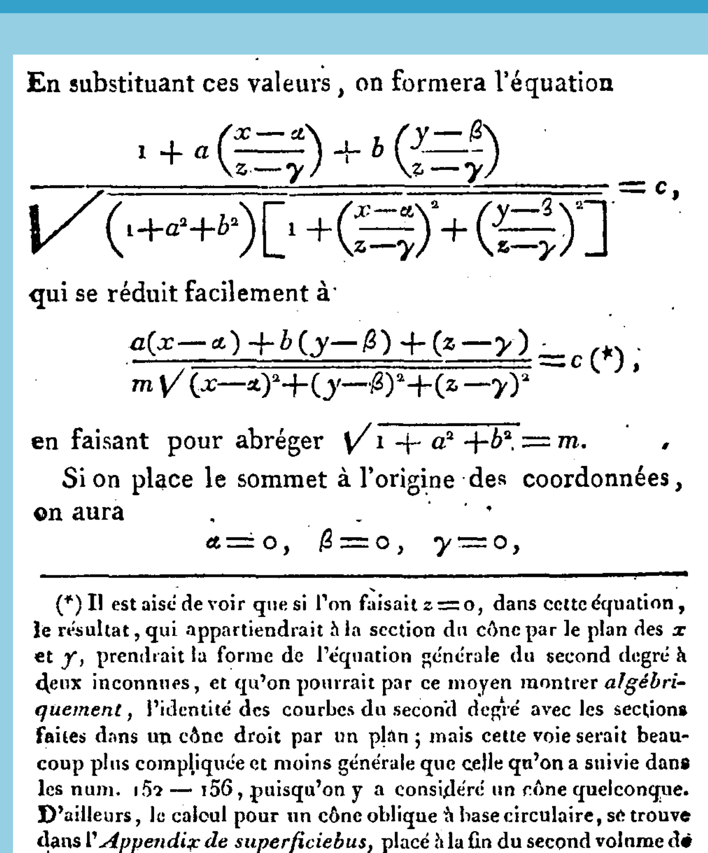


Any kind of cone cut by a plane

For **Menaechmus**, as for **Apollonius**, a parabola is the intersection of a (right) cone by a plane perpendicular to a generator of the cone. The difference between the two approaches comes from the angle at the apex of the cone; it is right angled for Menaechmus and any angle for Apollonius. In Menaechmus' conception, parabolas correspond to right angled cones, ellipses to acute angled and hyperbolas to obtuse angled. For Apollonius, the cone is fixed and it is the angle of inclination of the intersecting plane, with respect to the axis of the cone, which differentiates between the sections. The **parabola** is obtained from an angle equal to half the angle of the cone, the **ellipse** from a greater angle and the **hyperbola** from a smaller angle.



The equation of a cone
The conics are obtained for z=0



En substituant ces valeurs, on formera l'équation

$$\frac{1+a\left(\frac{x-\alpha}{z-\gamma}\right)+b\left(\frac{y-\beta}{z-\gamma}\right)}{\sqrt{(1+a^2+b^2)\left[1+\left(\frac{x-\alpha}{z-\gamma}\right)^2+\left(\frac{y-\beta}{z-\gamma}\right)^2\right]}}=c,$$

qui se réduit facilement à

$$\frac{a(x-\alpha)+b(y-\beta)+(z-\gamma)}{m\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}=c\ (*);$$

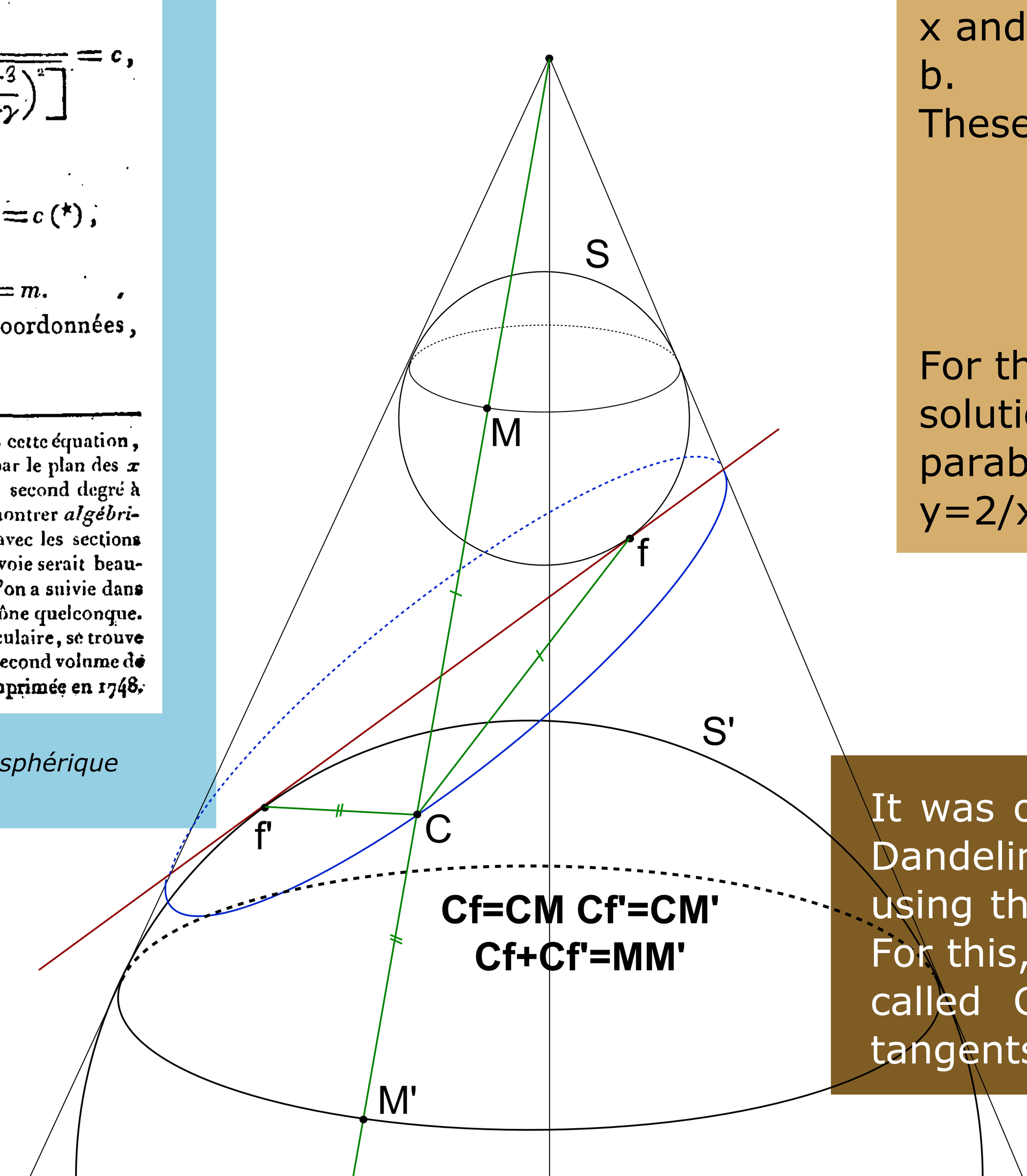
en faisant pour abréger $\sqrt{1+a^2+b^2}=m$.

Si on place le sommet à l'origine des coordonnées, on aura

$$\alpha=0,\quad \beta=0,\quad \gamma=0,$$

(*) Il est aisé de voir que si l'on faisait $z=0$, dans cette équation, le résultat, qui appartiendrait à la section du cône par le plan des x et y, prendrait la forme de l'équation générale du second degré à deux inconnues, et qu'on pourrait par ce moyen montrer *algébriquement*, l'identité des courbes du second degré avec les sections faites dans un cône droit par un plan; mais cette voie serait beaucoup plus compliquée et moins générale que celle qu'on a suivie dans les num. 152 — 156, puisqu'on y a considéré un cône quelconque. D'ailleurs, le calcul pour un cône oblique à base circulaire, se trouve dans l'*Appendix de superficiebus*, placé à la fin du second volume de l'*Introductio in analysin infinitorum* d'Euler, imprimée en 1748.

Traité de trigonométrie rectiligne et sphérique
S.Lacroix, 1807.

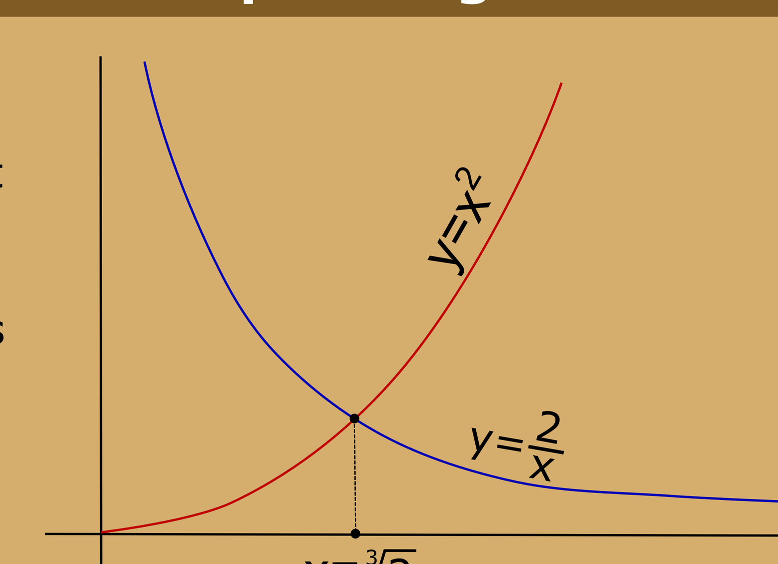


Graphical solution of Menaechmus for duplicating a cube

Given two magnitudes a and b of the same type, the problem is to find two other magnitudes x and y, of the same type as the previous ones, such that x and y are two mean proportionals between a and b.
These conditions translate into modern language as

$$a/x=x/y=y/b$$
$$x^3=a^2b \quad y=x^2/a \quad xy=ab$$

For the problem of Delos a = 1 and b = 2. The solution is obtained from the intersection of the parabola $y = x^2$ and the rectangular hyperbola $y=2/x$



It was only in the nineteenth century that two Belgian mathematicians, Dandelin and Quetelet, showed the equivalence of defining a conic by using the focii and defining it via the intersection of a plane and a cone. For this, spheres tangential to both the cone and plane are used. The so-called Gardener's definition turns up for the ellipse in tracing the tangents to the spheres.

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