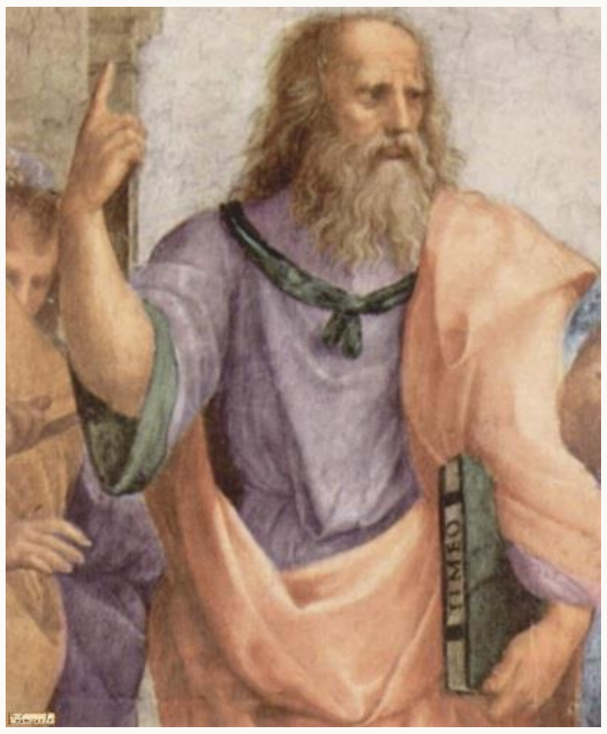
REPRESENTING

THE FIVE PLATONIC SOLIDS



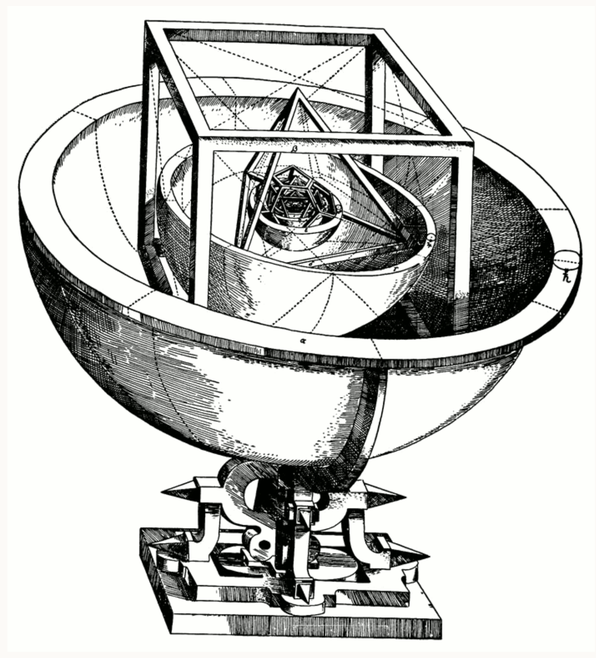
Plato: Detail from The School Of Athens by Raphael

«God used it for the Whole, when he designed the final arrangement.»
(Plato, Timeus 55a)

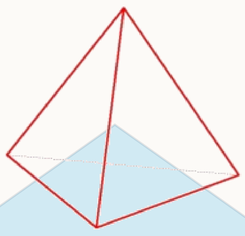
In search of regularity and harmony

From antiquity, Greek mathematicians have sought for geometric shapes corresponding to criteria of symmetry.

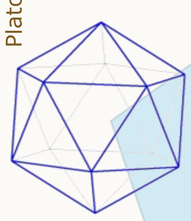
Theaetetus (-415, -395) appears to be the first theoretician of the **Platonic solids** called **convex regular polyhedra today**.



Model of the Solar System using models of the Platonic solids as viewed by Kepler in his Mysterium Cosmographicum (1596)

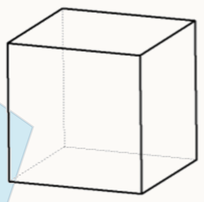


Tetrahedron
Fire / Jupiter
4 faces which are equilateral triangles



Icosahedron
Water
20 faces which are equilateral triangles

A polyhedron is called **regular** if all its faces are identical regular polygons, and if there is the same number of edges which converge at each vertex.



Cube
Earth / Saturn
6 faces which are squares

For PLATON (428-347 BCE) they were mythical objects

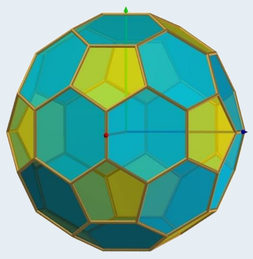
Euclid (325-265 BCE) gives, in The *Elements (Book XIII)*, a mathematical complete description of these solids. The proposals 13-17 describe the construction of: the tetrahedron, the octahedron, the cube, the icosahedron and the dodecahedron.
He gives as well the proof that there are only five.

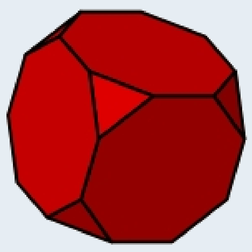
Dodecahedron
Whole / Mars
12 faces which are regular pentagons

Octahedron
Air / Venus
8 faces which are equilateral triangles

Kepler (1571, 1630) conjectured in the "***Mysterium Cosmographicum***" that these five regular solids and the spaces between the six planets known at that time were related. He thought that these regular solids were the key to understanding the architecture of the universe.

And if some constraints are relaxed ?



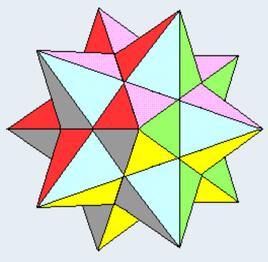


The Archimedean solids or semi-regular polyhedra

Truncated Icosohedron
32 faces

Truncated Cube
14 faces

Uniform Polyèdre: Small Dodecicosidodecahedron
18 faces

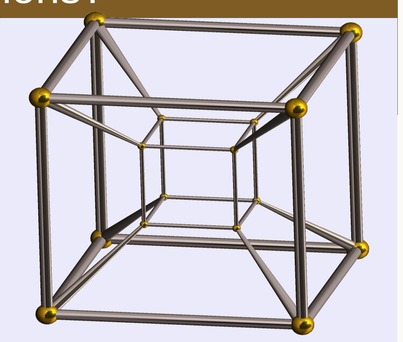


Kepler-Poinsot Polyhedra Stellatedodecahedron.
12 faces

And if we go into 4 or more dimensions?

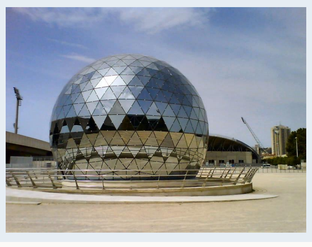
Example : the hypercube

In geometry, a **hypercube** is the n dimensional analogue of a square (*n=2*) and of a cube (*n = 3*). It is a closed, compact, convex shape consisting of opposite groups of parallel segments aligned in each dimension of the space and at right angles relative to each other.



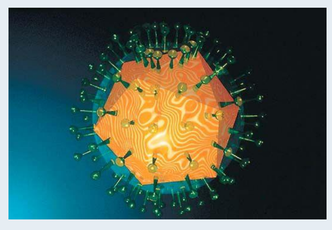
Polyhedra around us

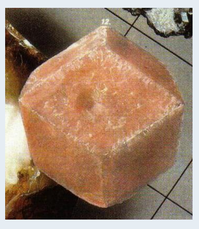
Football
Truncated Icosahedron



The geode on the esplanade at Ganay

Herpes Virus
Regular Icosahedron





Garnet crystal
Rhombic dodecahedron

IREM Aix-Marseille
http://www.irem.univ-mrs.fr/expo2013