# STORIES OF SPHERES

LOCATING

## Problems of astronomy

The celestial vault is represented by a sphere. The heavenly bodies, stars or planets, are known by their celestial coordinates, generally by their right ascension, the equivalent of longitude which is measured relative to a reference meridian and is expressed in hours, minutes, seconds, and by their declination, the equivalent of latitude, which is measured from the plane of the horizon.

Astronomers have questions like:
- What is the angle between two stars, known by their coordinates?
- What is the angular distance traveled by a planet in a given time?

## Problems of navigation

The Earth is likened to a sphere. We represent latitude by the angle from north or south of the equator and longitude, by the angle relative to the Greenwich Prime Meridian, East or West.

Navigators have two fundamental questions:
- What is the distance between two points known by their latitude and longitude?
- What course should one follow? (the angle between the route of the boat and the North)

## The calculation of orthodromic distances

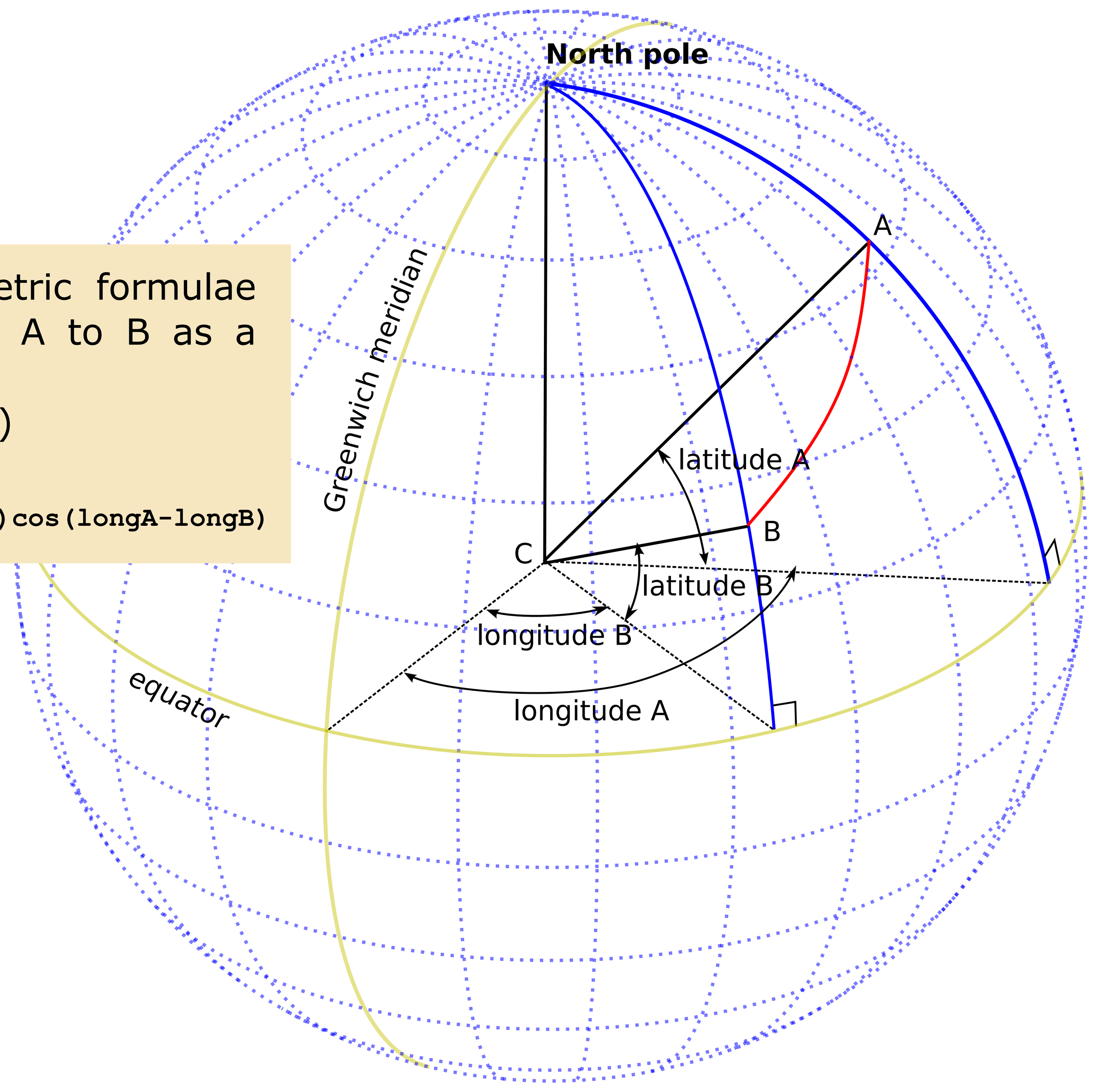
the shortest path from point A to point B on the Earth's surface

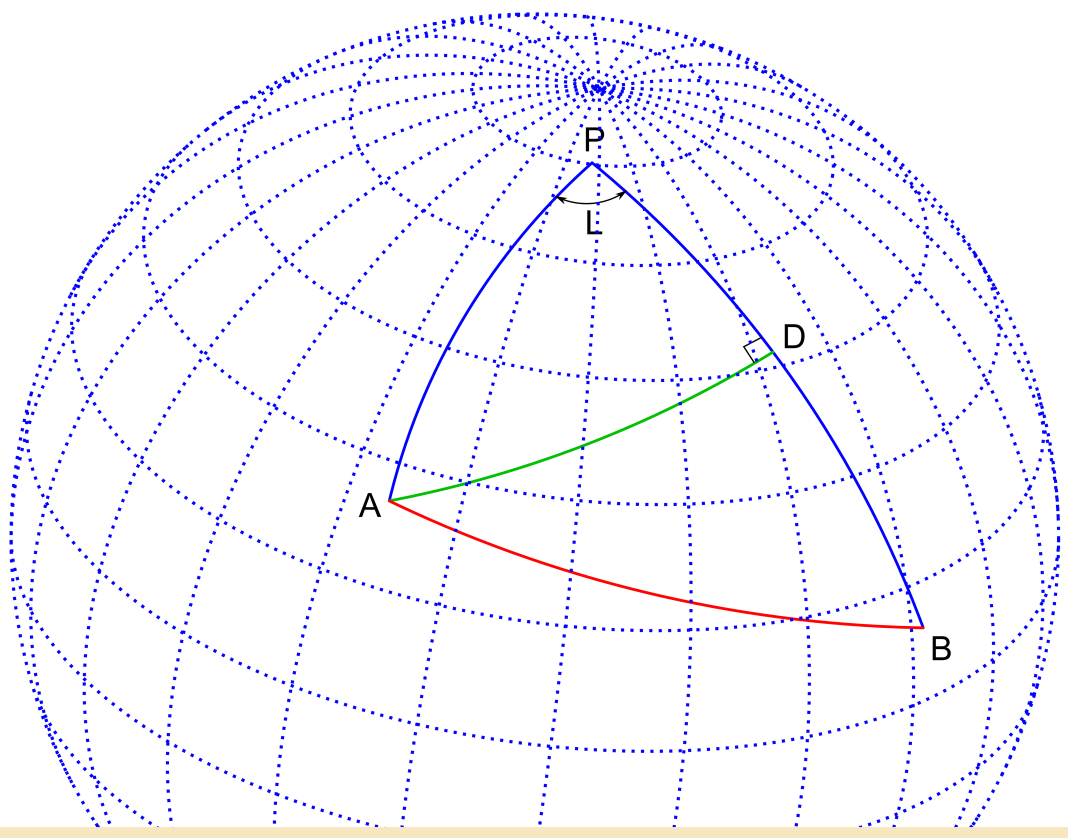
From the 19th century on, trigonometric formulae have directly given the distance from A to B as a function of latitude and longitude.

AB = radius x angle (ACB)

The ACB angle is given by the formula:

$$\cos(ACB)=\sin(latA)\sin(latB)+\cos(latA)\cos(latB)\cos(longA-longB)$$



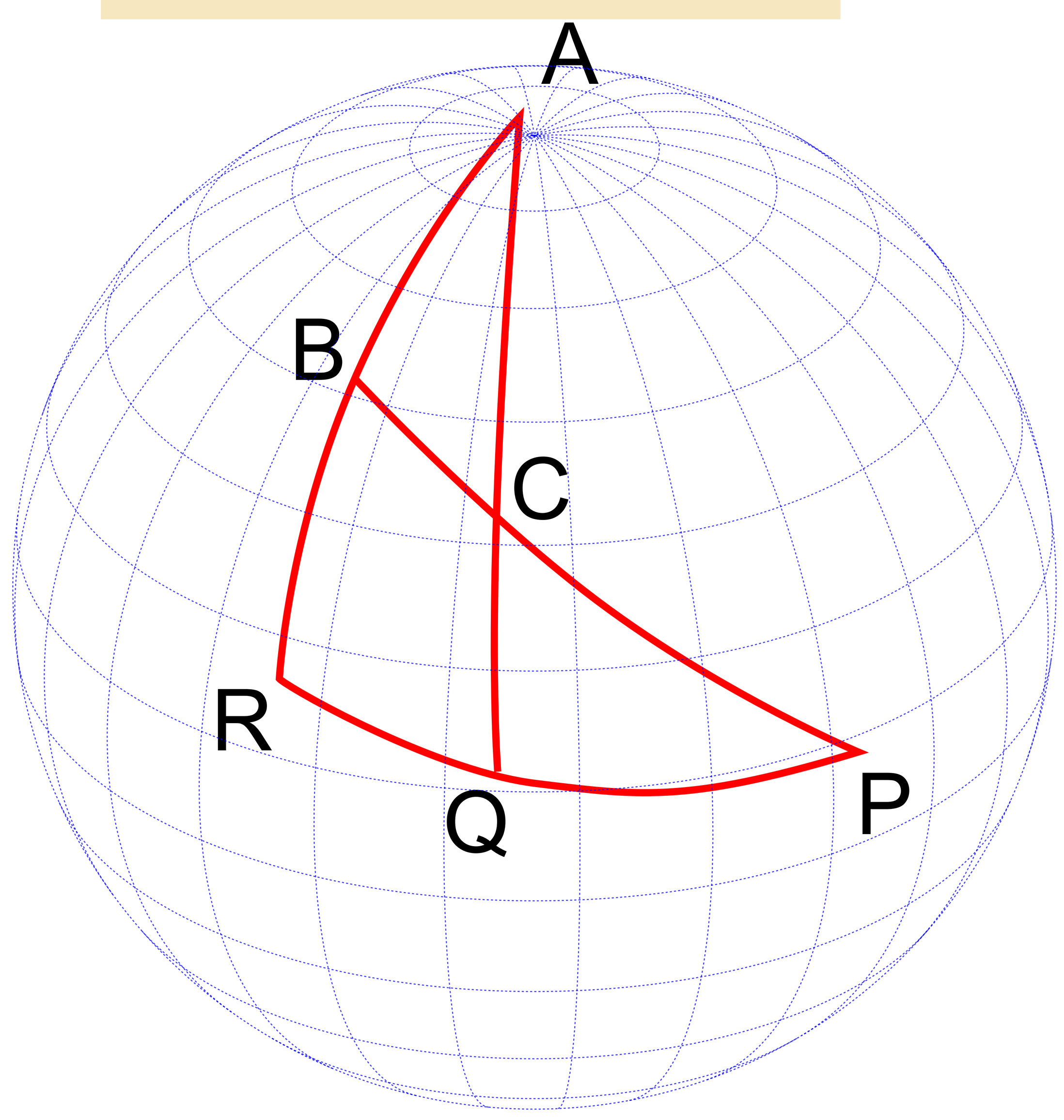

From the 16th to 18th century, navigators and astronomers mainly used the decomposition into right angled spherical triangles for which the calculations were easier. The appearance of Napier's logarithms facilitated these calculations which include formulae of the type cos (a) = cos (b) cos (c).
PAB any triangle, to obtain AB calculate AD and DP successively in the triangle DAP, then DB, and AB in the right angled triangle DAB.

## The theorem of Menelaos

ABR, BCP, ACQ, great circle arcs on a sphere of radius 1.

great circle arcs are equated with central angles

$$\frac{\sin(PB)}{\sin(PC)} \times \frac{\sin(QC)}{\sin(QA)} \times \frac{\sin(RA)}{\sin(RB)} = 1$$




From Antiquity to the Renaissance, the **Theorem of Menelaus** was the basis of all calculations on the sphere, terrestrial or celestial. Ptolemy developed astronomical applications in the Almagest, which have remained valid and were used for over a millennium. This theorem was perhaps known to Hipparchus, but was attributed to Menelaus in his work the "Sphaerica" which was translated into Syriac, Arabic and Latin from Greek

## The dissemination of the "Sphaerica" of Menelaos