MEASURING
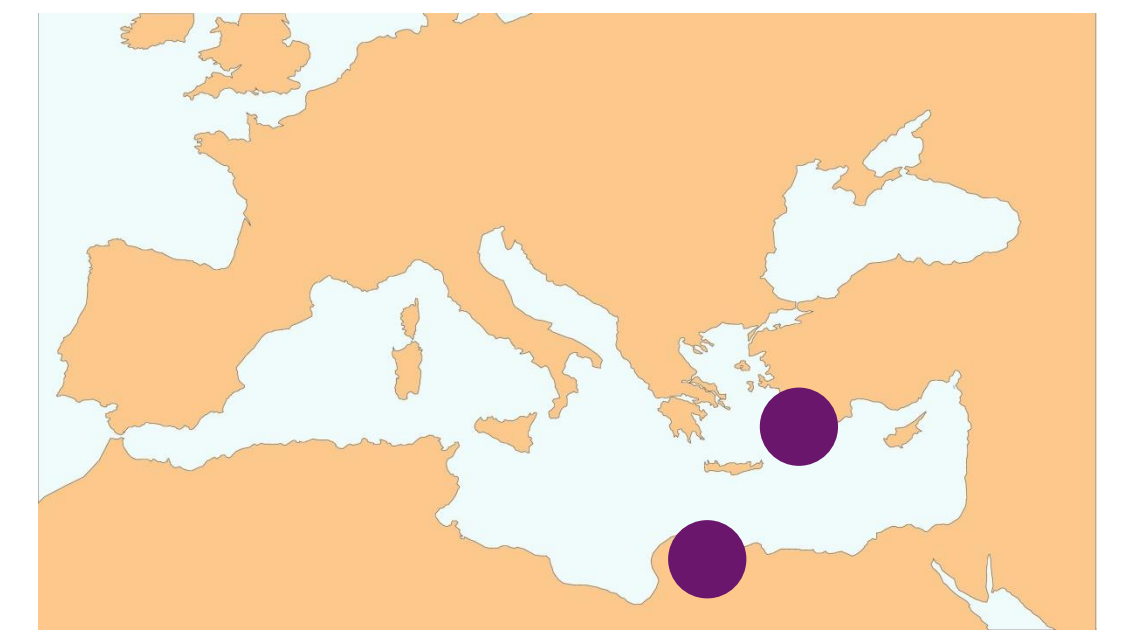
MEASURING THE EARTH AND THE HEAVENS
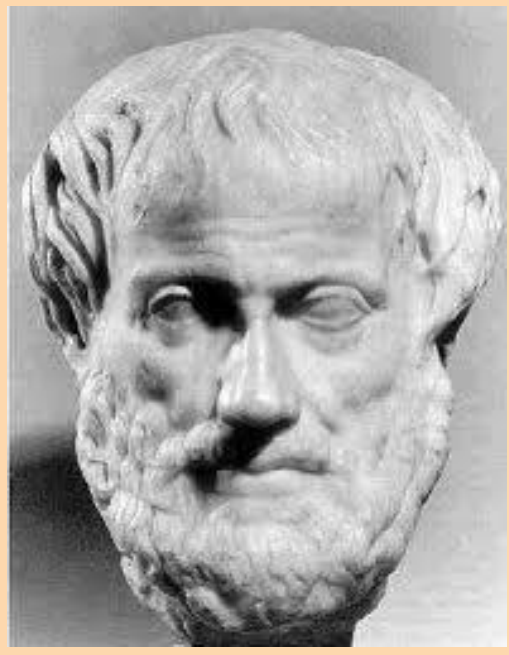# ARISTARCHUS AND ERATOSTHENES

17 CENTURIES BEFORE COPERNICUS, ARISTARCHUS HYPOTHESISED THAT IT IS THE EARTH WHICH REVOLVES AROUND THE SUN AND ERATOSTHENES EVALUATED THE RADIUS OF THE EARTH TO WITHIN 2%.

ARISTARCHUS CALCULATED THE DISTANCES AND THE RADII OF THE MOON AND THE SUN

-310 to -230 Samos

To calculate the Earth-Moon and Earth-Sun distances, he make these assumptions :

1. The moon receives light from the sun.
2. The Earth can be considered as the centre of the orbit of the Moon.
3. When the Moon appears to us "dikhotome" (cut in two equal portions), its great circle is apparent to our eyes, determining the illuminated and the dark parts of this celestial body.
4. When the Moon appears to us "dikhotome", its distance from the Sun is less than a quarter of the circumference by the thirtieth part of this quarter.
5. The width of the shadow (of the Earth during a lunar eclipse) is two Moons.
6. The arc subtended in the sky by the moon is the fifteenth part of a sign (of the zodiac, so 2°).
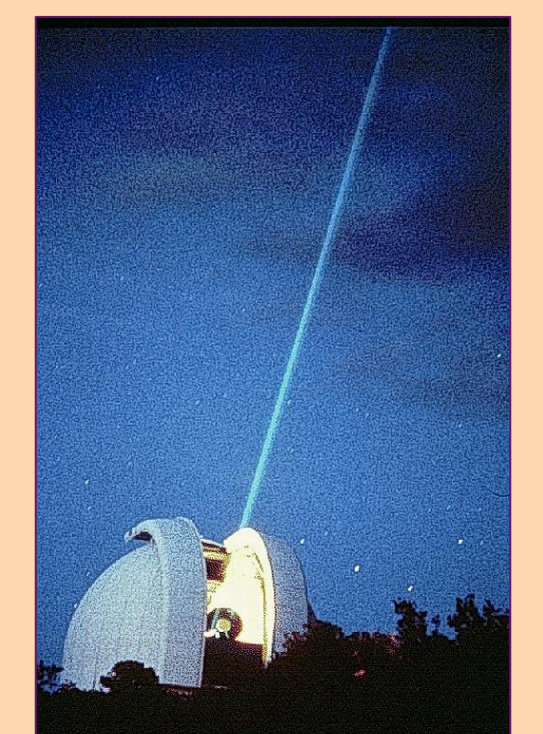
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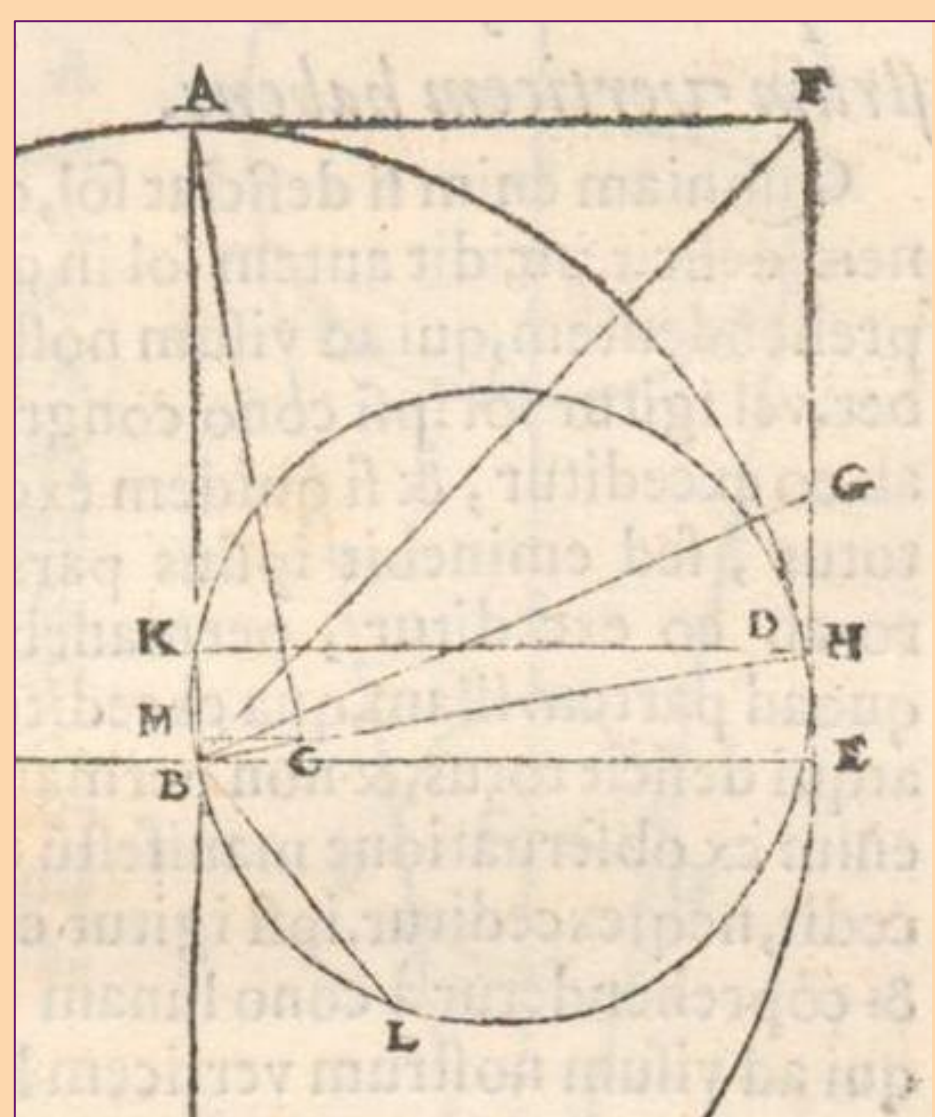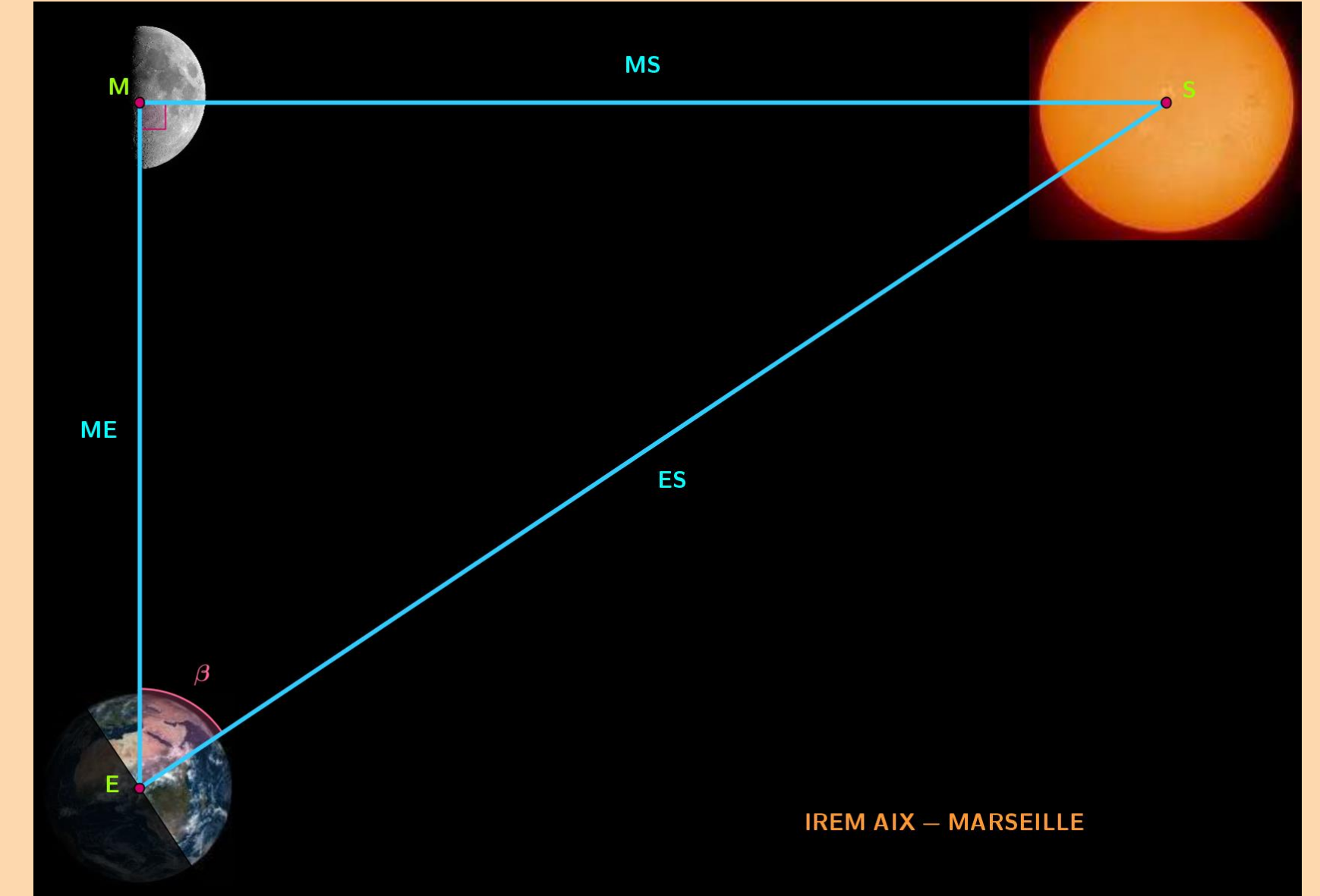
Without any trigonometry (which only appeared with Hipparchus 100 years later), Aristarchus then deduced, in only considering relationships of length that :

✓ the lunar diameter is less than 2/45 and greater than 1/30 of the Earth-Moon distance.
   Actual value: 0.09 is about 4 times less.

✓ the solar diameter is less than 43/6 and greater than 19/3 of the earth's diameter.
   Actual value: 112 is about 17 times more.

✓ the Earth's diameter is less than 60/19 and greater than 108/43 of the lunar diameter.
   Actual value: 3.67 is about 2.6 times more.

Hypothesis 4 amounts to giving the angle β the value of 87 ° (instead of 89.85 °) and this difference explains the error in his calculation of the Earth-Sun distance.
Hypothesis 6 gives the Moon a diameter of 2 °, four times the actual value.

« The diameter of the Sun to the diameter of the Earth is in a greater proportion than 19 to 3 and in a lesser by 43 to 6 »

The Sun being larger than Earth, as the Earth is larger than the moon, he had the intuition that it is surely the Earth that revolves around the Sun!
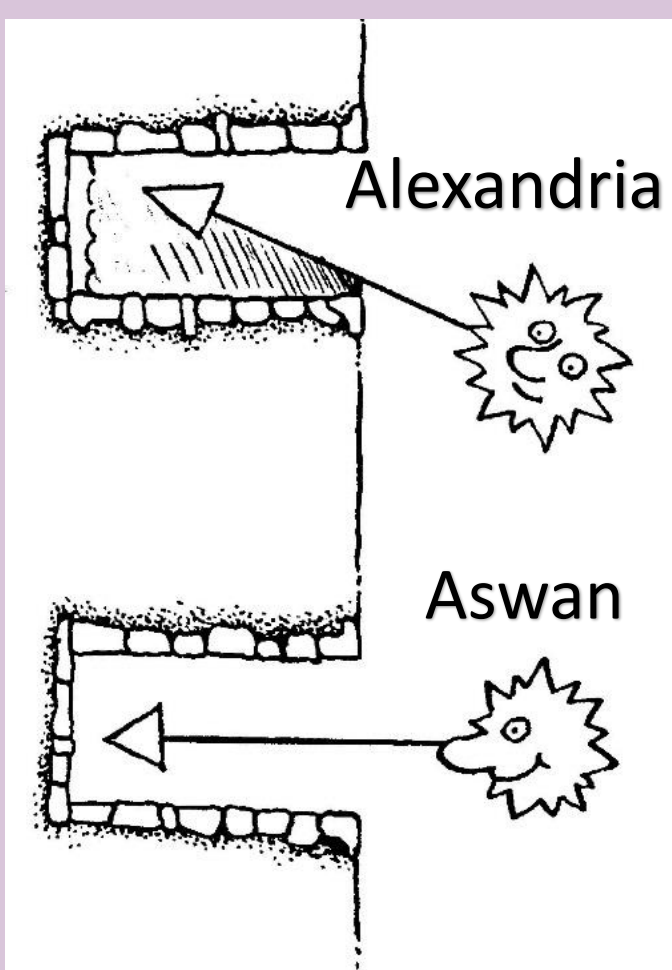He is therefore the author of the first heliocentric system of the world.

Lunar eclipse (june 2011)

Measuring the Earth-Moon distance today, by laser to within 4 mm .
(ie a relative accuracy of 10^{-13})

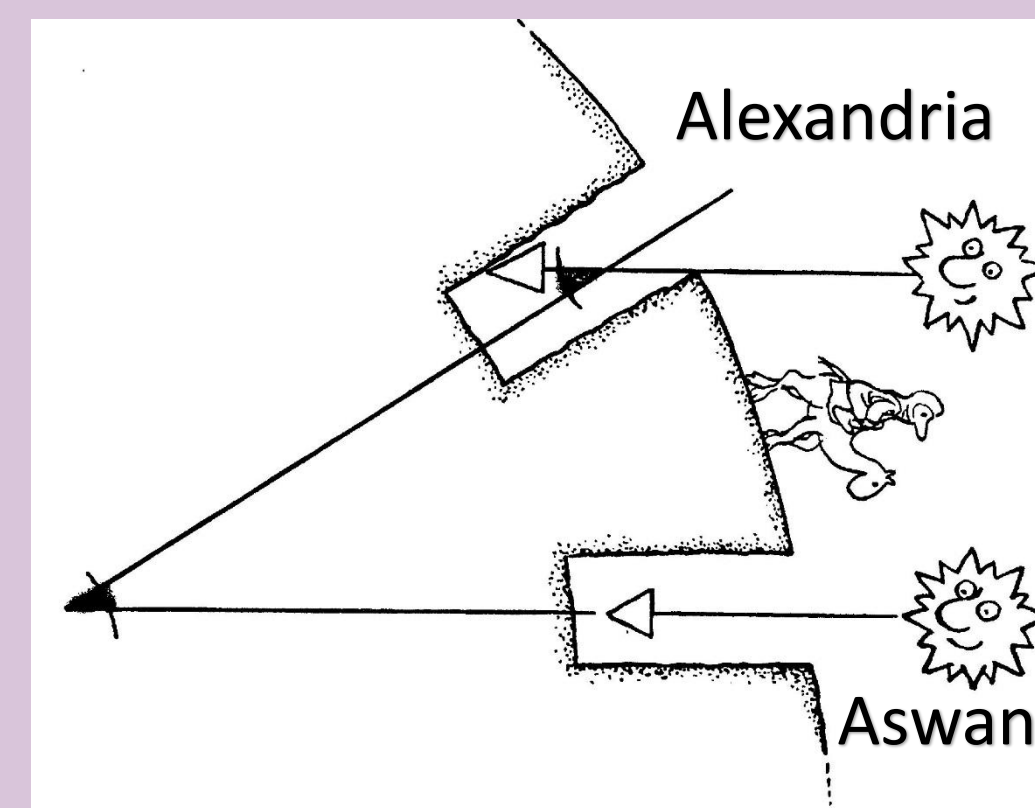## ERATOSTHENES MEASURES THE RADIUS OF THE EARTH

He noticed that at noon on the day of the summer solstice at Syene (Aswan today, situated below the Tropic of Cancer) objects cast no shadow and that the Sun can be observed at the bottom of a well. This phenomenon did not take place in Alexandria, further north on the same day.

he envisages two hypotheses :
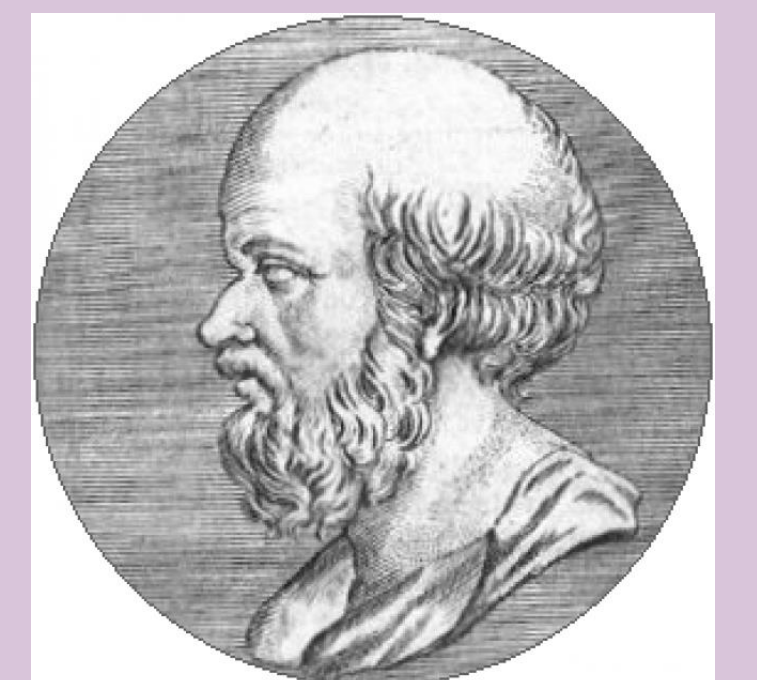
-274 Cyrene to -194 Alexandria

Alexandria
Aswan

Either the Earth is flat and the Sun's rays are not parallel, which implies a very close Sun.

Or the Sun is far enough away that its rays arrive parallel to Earth, but then the Earth is not flat.

Alexandria
Aswan

Two hundred years before him, Anaxagoras had chosen this hypothesis (flat Earth): he then arrived at the result that the Sun was about 6500 km from Earth and had a diameter of .... 60 km!

The estimated distance from the Sun allowing him to set the first hypothesis aside, he continued by measuring the angle **α** between a vertical gnomon and the direction of the Sun's rays in Alexandria. **Value : 7,2°.**

Alexandria — *Gnomon(stick)*
Sun rays
A — α
Aswan
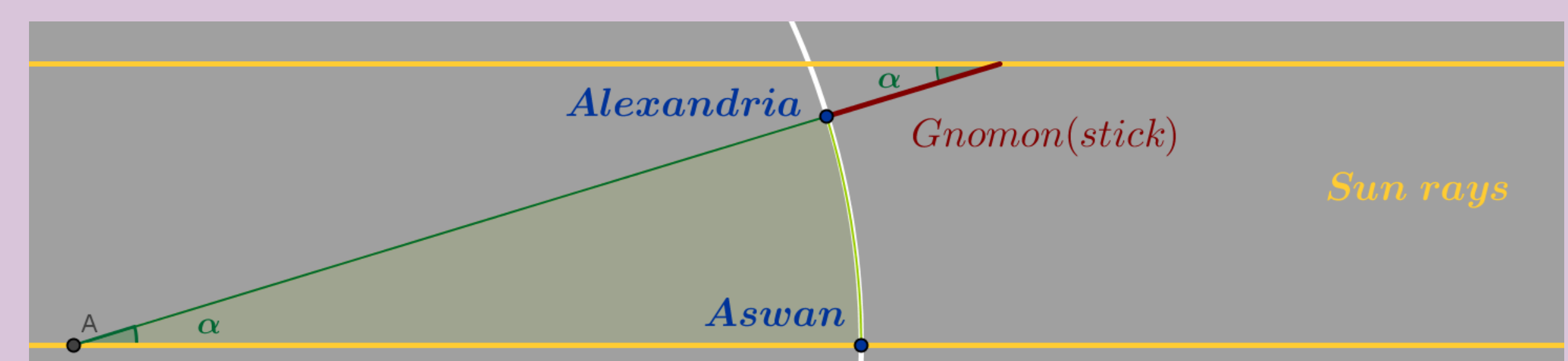
He then measured the distance between the two cities by counting the number of days of travelling by camel. The result was miraculously precise: 787 km instead of 800 km.
A simple calculation using proportionality then allowed him to calculate the circumference of Earth.
He then deduced a measurement of the radius which was an excellent approximation :
6266 km instead of 6378 km

This measurement remains a symbol of the ingenuity that allows a curious, observant mind to obtain a fundamental result by very elementary means.