MEASURING

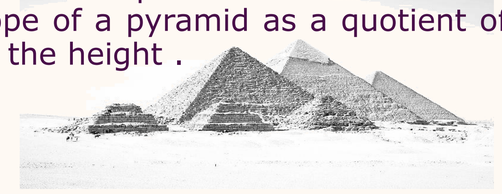
THE MEDITERRANEAN ORIGINS OF TRIGONOMETRY

TRIGONOMETRY = Tri + Gono + Metry
Mesuring three sides

FIRST APPEARENCES

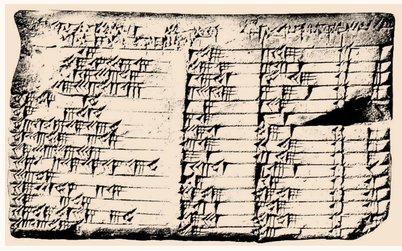
IN ANCIENT EGYPT (circa. 4000 B.C.)

Problems R56 to R59 of the Rhind papyrus give a detailed explanation of the method of calculating the slope of a pyramid as a quotient of the half of the base by the height .



THE BABYLONIANS (circa 1600 B.C.)

The Babylonian astronomers calculated the ratios of sides of similar triangles. They introduced the division of the circle into 360 degrees.



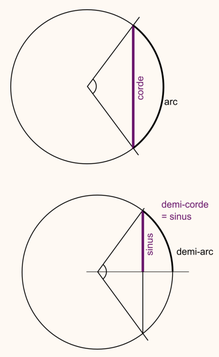
Plimpton 322 tablet contains a list of Pythagorean triplets. They are tidied up in increasing order, with regard to the quotient of the first two numbers

IN ANCIENT GREECE (2th century B.C. - 2th century A.D.) Astronomer mathematicians

Hipparcus of Nicea (190 B.C., 120 B.C.)

First « chord tables »

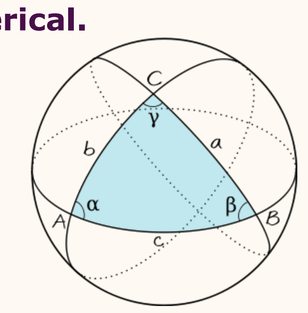
He was the first to establish these (lost, but known to Ptolemy and Theon of Alexandria). These "**chord tables**" were useful for calculating the eccentricity of the lunar and solar orbits, or in the calculations of the magnitudes and distances of the Sun and the Moon.
With the aid of these tables, he discovered that the axis of the Earth is not fixed!
He improved the calculation of the distance from Earth to the Moon. As the Babylonians once did, he introduced the division of the circle into 360 degrees, dividing the degree into 60 minutes, and the minute into 60 secs.



Menelaus (70, 140)

And trigonometry became spherical.

His books (*Chords in the circle, Sphaerica*) deal with the geometry of the sphere and its applications to astronomy. He defines the spherical triangle in them.

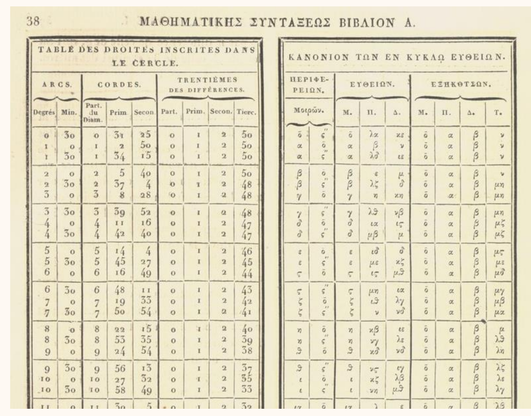


The Almagest, manuscript 1020 in Hebrew, p11, BNF

Claudius Ptolemy (90, 168)

The Almagest is a reference work (thirteen books) of mathematical astronomy.

Ptolemy reconstructed all the trigonometry of antiquity in the Almageste, for use in the chapters on Astronomy. He explained how to calculate the lengths of chords and published a comprehensive table of them, thus improving those of Hipparchus. It was at this time that the Greeks took up the practice of dividing the circle into 360 degrees, as Hipparchus had done.
He also developed tools for spherical geometry.
His chord tables would be one of the initial approaches in the history of mathematics, leading to the concept of functions.
His texts have been used over more than 1300 years, right up to Nicolas Copernicus.



Ptolemy's chord table, BNF

***House of Wisdom* at BAGDAD**
Indian and Persian astronomical treatises are found there.

IN THE ARABO-MUSLIM WORLD, 8th century – 13th century Astronomer mathematicians

SINE
Indians replaced in 6th century, the notion of chord by a notion called later "sine".

Al Khwarīzmī (783 - 850)

The father of algebra, but an astronomer as well, he was influenced by Aryabhata (Indian astronomy) and also by Persian and Greek astronomy. He refined Ptolemy's chord tables in his *zîj*, providing a "sine" table of Indian provenance. He made his contribution to spherical trigonometry and also wrote books on the instruments such as the astrolabe.

Manuscript of the text of Al-Kharizmi at the end of the 14th century, gathering diverse astronomical treaties, preserved in Staatsbibliothek of Berlin

Habash Al Hāsib (770 - 870)

Nicknamed the « calculator ». The sine and versed sine (1 - cos) are defined in his *zîj* (table). He also defined the notion of a tangent and established a table, but it went unnoticed.

Universal astrolabe of Iraq, 1210, Pergame Museum, Berlin

Abū'l-Wafā (940 – 998)

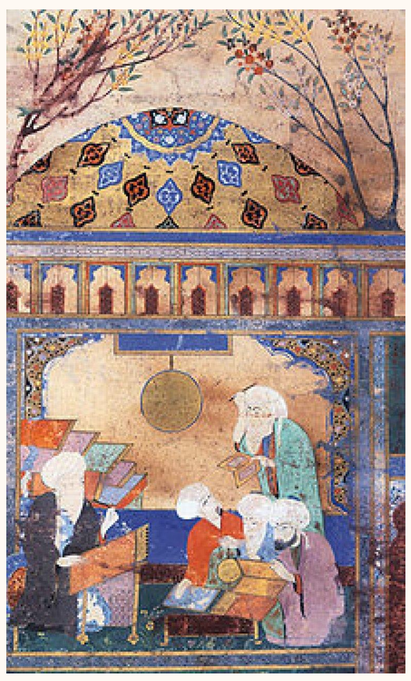
He recognized the importance of the tangent. In *The Revision of the Almagest*, he completed the trigonometric tables of his predecessors. We owe the concept of the trigonometric circle of radius 1 to him.

Nasîr Al Dîn Tûsî (1201- 1274)

In the *Treatise on the Quadrilateral* , he makes a synthesis of previous treatises. He is considered the first to treat trigonometry as a distinct mathematical discipline.

He establishes the Sine Rule for plane triangles.

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$



Nasir al Din al Tusi at his desk at the Maragha observatory, reading measurements on an astrolabe. 15th century Persian manuscript, No. 1418 at the University of Istanbul

Regiomontanus (1436-1476)

De triangules omnismodis is the culmination of the construction of this discipline and the beginning of trigonometry as a entirely separate discipline.

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