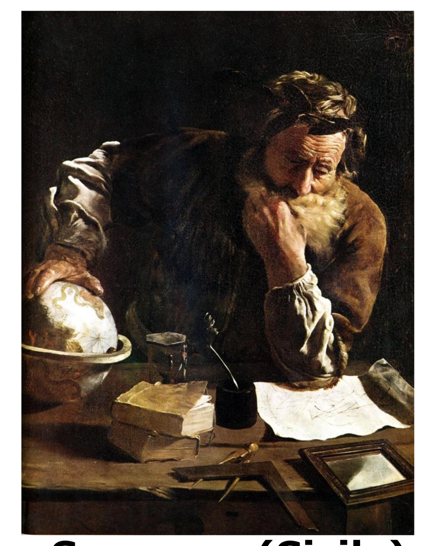
MEASURING

AT THE ORIGINS OF INTEGRAL CALCULUS
ARCHIMEDES

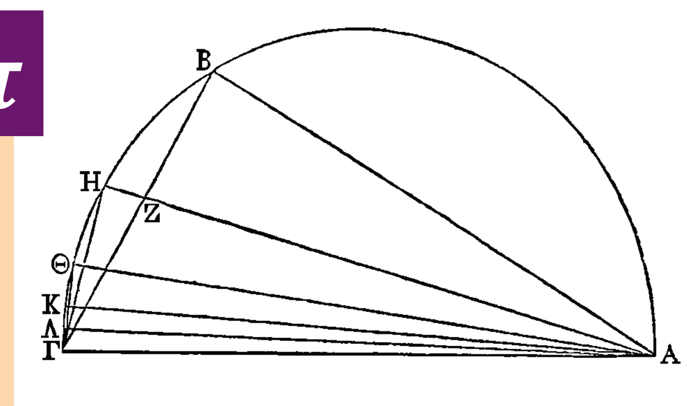


Syracuse (Sicily)

approx -287 to -212 BCE

GENERALLY CONSIDERED, WITH EUCLID, AS THE GREATEST MATHEMATICIAN OF ANTIQUITY, HIS METHODS PREFIGURE THOSE OF THE INTEGRAL CALCULUS WAS TO BE DEVELOPED BY NEWTON AND LEIBNIZ IN THE 7TH CENTURY AND WHICH HAVE MADE ENORMOUS ADVANCES IN MATHEMATICS AND ITS APPLICATIONS POSSIBLE.

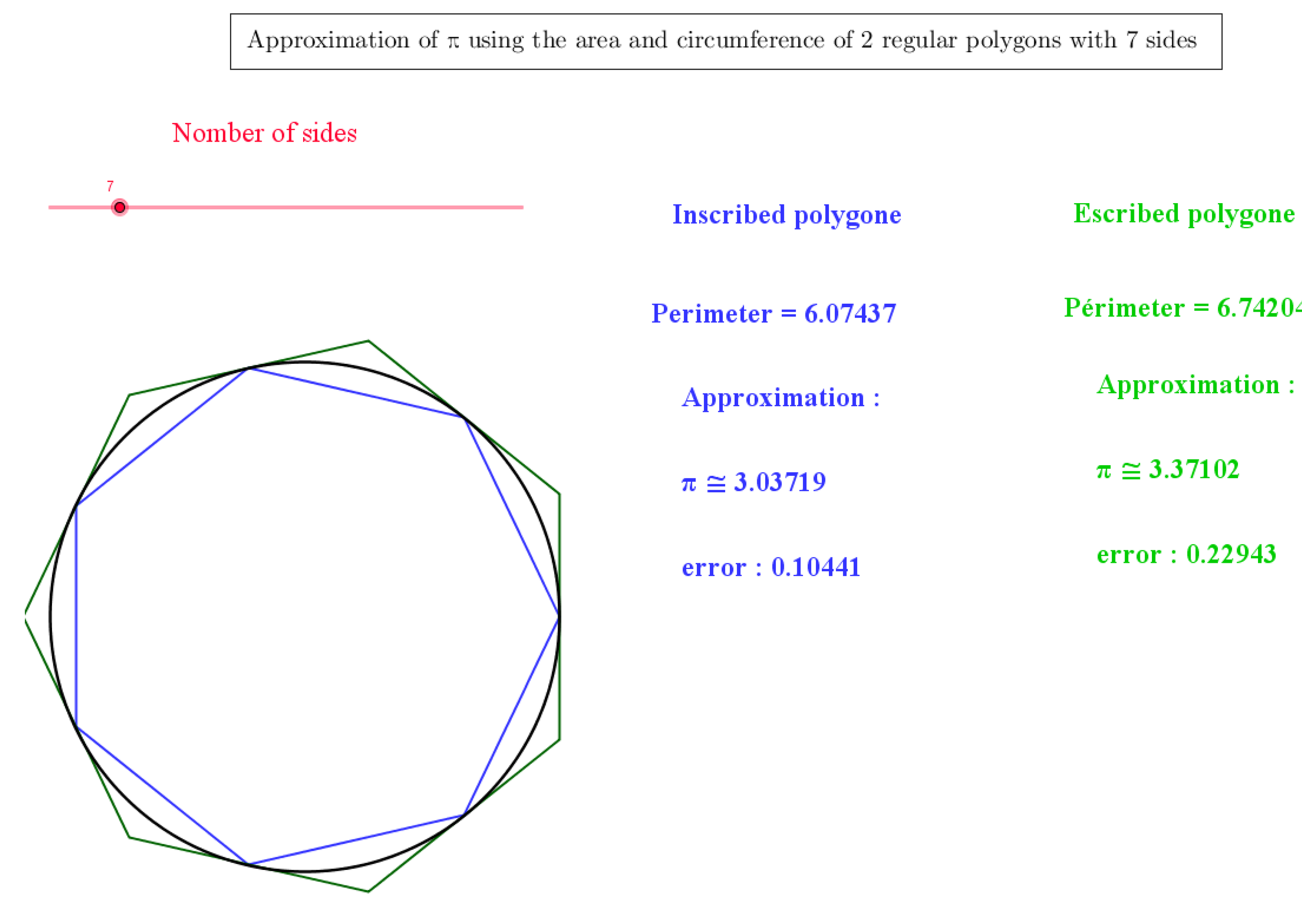
MEASUREMENT OF π



Proposition III of the Treatise "On the Measurement of the Circle":

“ The circumference of any circle is equal to the triple of the diameter combined with a portion of the diameter, which is smaller than the seventh of this diameter, and greater than 10/71 of the same diameter ”

Archimedes invented the following method, circa 250 BCE, to calculate the length of the perimeter of a circle: he sets bounds on this value with the perimeter of a regular polygon inscribed in the circle, and the perimeter of an escribed regular polygon.



Using a polygon with 96 sides (he starts with a hexagon and multiplies the number of vertices four times by two), Archimedes arrives at the limits :

$$3+\frac{10}{71}<\pi<3+\frac{1}{7}$$ **which gives** $$\frac{223}{71}<\pi<\frac{22}{7}$$ **(amplitude 0,002)**

The use of the Greek letter π (p for perimeter) dates only from the 18th century !

The method of exhaustion, sketched out by Eudoxus and formalised by Euclid, is applied by Archimedes to several problems of calculation of lengths, areas and volumes.

Archimedes’ screw

His works are concerned not only with arithmetic and geometry, but also with mechanics (worm bears, levers), hydrostatics ("any body immersed in a liquid ...."), etc .

« Eureka »

VOLUME OF A SPHERE

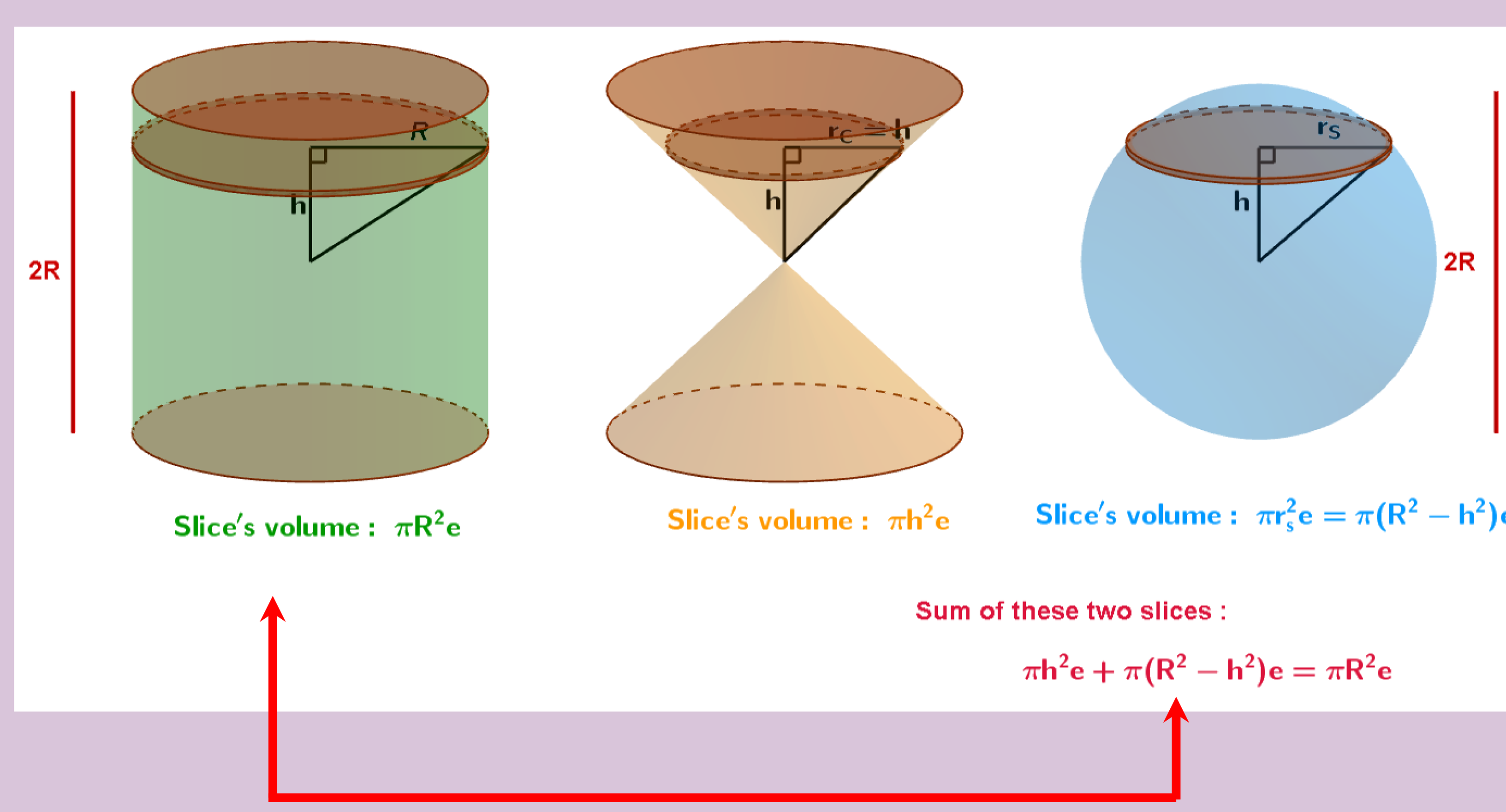
Archimedes shows that the volume of the cylinder to the left (of height 2R) is the sum of that of the sphere and the right cone.

- ✓ **Each volume is cut into thin horizontal slices of thickness e.**
- ✓ **This value, e, can be as small as you like and the total volume is the sum of these slices.**

As we always have (for any h) : $R^2 = h^2 + {r_S}^2$, **thus we deduce, for each slice :**

$T_{cylinder} = T_{hourglass} + T_{sphere}$ **and therefore for the entire volume**

$$V_{cylinder} = V_{hourglass} + V_{sphere}$$

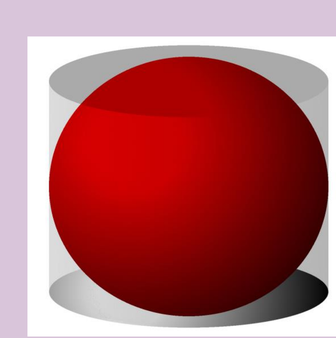


As he knows that the hourglass has a volume equal to 1/3 that of the cylinder, he deduces from this :

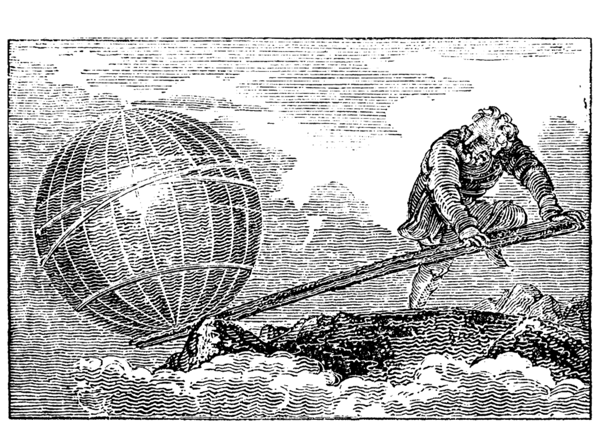
$V_{sphere} = \frac{2}{3} V_{cylinder} = \frac{2}{3} 2\pi R^3$, **or :**

$$V_{sphere} = \frac{4}{3}\pi R^3$$

Archimedes was so proud of this discovery he would have given instructions for his tomb to be engraved with a sphere inscribed in a cylinder



« Give me a fulcrum and a lever and I will lift the World »



Despite (or because of) the originality of his works, Archimedes was not greatly followed in the Greek world and it was not until the 9th century that the Arabs (especially Ibn Qurra) translated his treatises and exploited his methods.

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