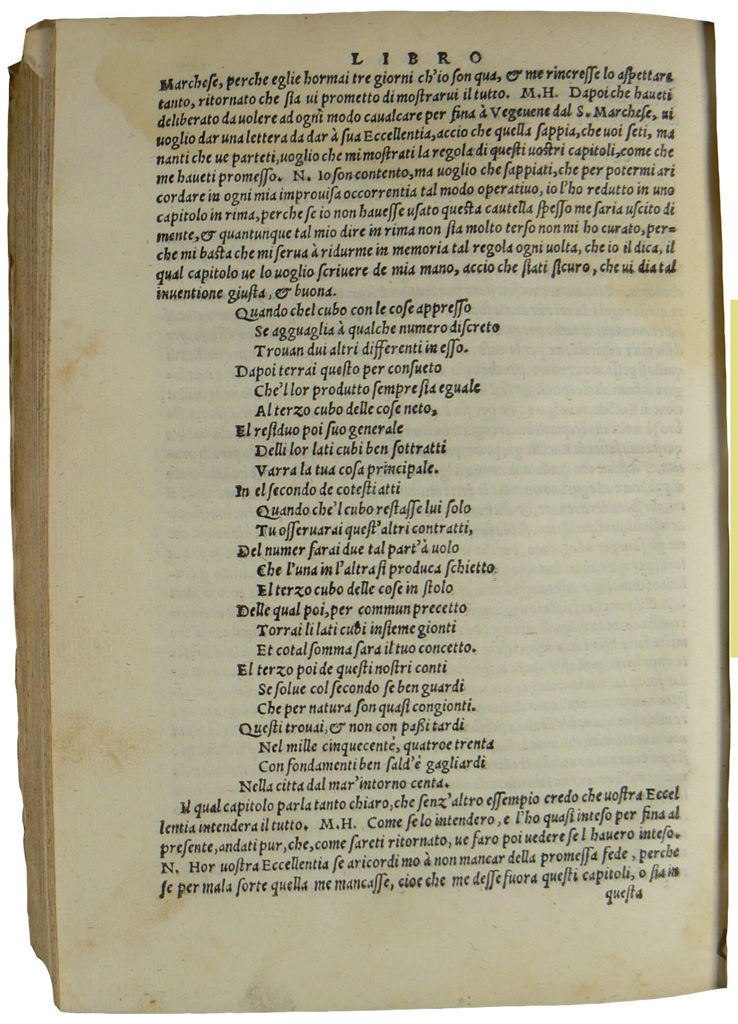
EMERGENCE

THE RENAISSANCE OF MATHEMATICS

TARTAGLIA AND CARDAN

Tartaglia published his method for the solution of third degree equations in the QUESTI in the form of a poem



Extract from "Quesiti et inventione diverse", First edition(Venise, 1554). Alcazar Library, Marseille.

When the cube with the things
Is equal to any discrete number
Find two numbers differing in this
Then you will customarily take
Their product to be always equal
To the cube of the third of the things exactly.
Next the remainder as a general rule
Of their cubic roots properly subtracted
Will be equal to the principal thing.
In the second of these acts
When the cube remains alone
You will observe these other contracts
You will divide the number in two parts
Such that the one times the other
clearly produces
The cube of the third of the things exactly.
Next from this by an habitual rule
You extract the cubic roots added together
This sum will become your principal result.
Next the third of our calculations
Is solved with the second if you take care
Since by their nature they are almost linked.
I have found these things not slowly
In one thousand five hundred and thirty four
With strong and sure foundations,
In the city surrounded by the sea

English translation of the solution of equations of the third degree

x^3+px

$=q$

$q=u-v$

$uv=$

$(p/3)^3$

$\sqrt[3]{u}-\sqrt[3]{v}$

$x^3=px+q$

$q=u+v$

$uv=$

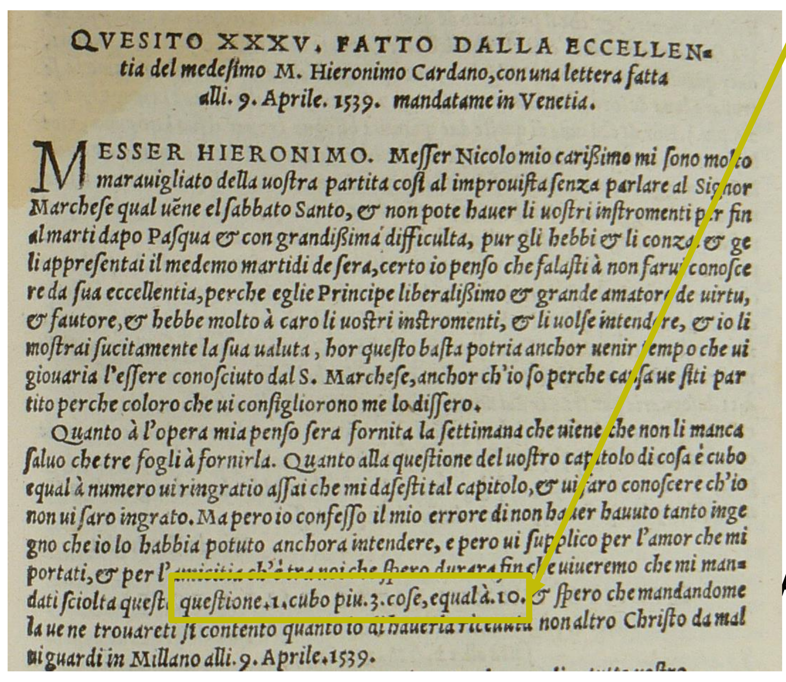
$(p/3)^3$

$\sqrt[3]{u}+\sqrt[3]{v}$

$=x$

$x^3+q=px$

Translation into modern algebraic language



Qvesito XXXV. fatto dalla eccellentia del medesimo M. Hieronimo Cardano, con una lettera fatta alli. 9. Aprile. 1539. mandatame in Venetia.

Messer Hieronimo. Messer Nicolo mio carissimo mi sono molto marauigliato della uostra partita cosi al improuista senza parlare al Signor Marchese qual uene el sabbato Santo, & non pote hauer li uostri instromenti per fin al marti dapo Pasqua & con grandissima difficulta, pur gli hebbi & li conzai & ge li appresentai il medemo martidi de sera, certo io penso che falasti à non faruí conoscere da sua eccellentia, perche eglie Principe liberalissimo & grande amatore de uirtu, & fautore, & hebbe molto à caro li uostri instromenti, & li uolse intendere, & io li mostrai sucitamente la sua ualuta, hor questo basta potria anchor uenir tempo che ui giouaria l'essere conosciuto dal S. Marchese, anchor ch'io so perche causa ue siti partito perche coloro che ui consigliorono me lo dissero.

Quanto à l'opera mia penso sera fornita la settimana che uiene che non li manca saluo che tre fogli à fornirla. Quanto alla questione del uostro capitolo di cosa è cubo equal à numero ui ringratio assai che mi dasesti tal capitolo, & ui faro conoscere ch'io non ui saro ingrato. Ma pero io confesso il mio errore di non hauer hauuto tanto ingegno che io lo habbia potuto anchora intendere, e pero ui supplico per l'amor che mi portati, & per l'[illegible] fin che uiueremo che mi mandati sciolta questa questione.1.cubo piu.3.cose, equal à.10. & spero che mandandome la ue ne trouareti [illegible] non altro Christo da mal ui guardi in Millano alli. 9. Aprile. 1539.

Extract from Quesiti XXXV

1 cube plus 3 things equals 10

Return to solve the equation
$x^3+3x=10.$

p = 3 and q = 10

Let u – v = 10 and uv $=\left(\frac{3}{3}\right)^3 = 1$

Then we have u-v = 10 and
v = $\frac{1}{u}$ let u - $\frac{1}{u}$ = 10
Then u is the solution of
$u^2 - 10u - 1 = 0$

The value of the discriminant is 104
And the positive solution of this equation is :
u $=\sqrt{26}+5$ et donc v = $\sqrt{26}-5$

Thus a solution to the given problem is

x = $\sqrt[3]{\sqrt{26}+5}-\sqrt[3]{\sqrt{26}-5}$

NICCOLO FONTANA called TARTAGLIA (1499-1557)

Niccolo Fontana was born in Brescia. During the sack of this city by the French (1512), his jaw was split by a sword. This left him with a speech difficulty which earned him the nickname Tartaglia (stutterer).

In 1534, he moved to Venice as a professor of mathematics.
In 1535, during a confrontation with Antonio Maria Fior (a student of the mathematician Scipione del Ferro), he was asked to solve thirty third degree equations of type.

$x^3 + px = q$

Tartaglia solved the equations within hours, just before the deadline. Furthermore it was only for the honour, as he gave up the prize of thirty successive banquets.

In the hope of winning other contests, Tartaglia did not reveal his formula. He then maintained a long correspondence with Cardan on the subject of his formula.

Tartaglia published the *Quesiti and diverse Inventioni* in 1546 after the work of Cardan. His latest book, *General Trattato* was not published until after his death.

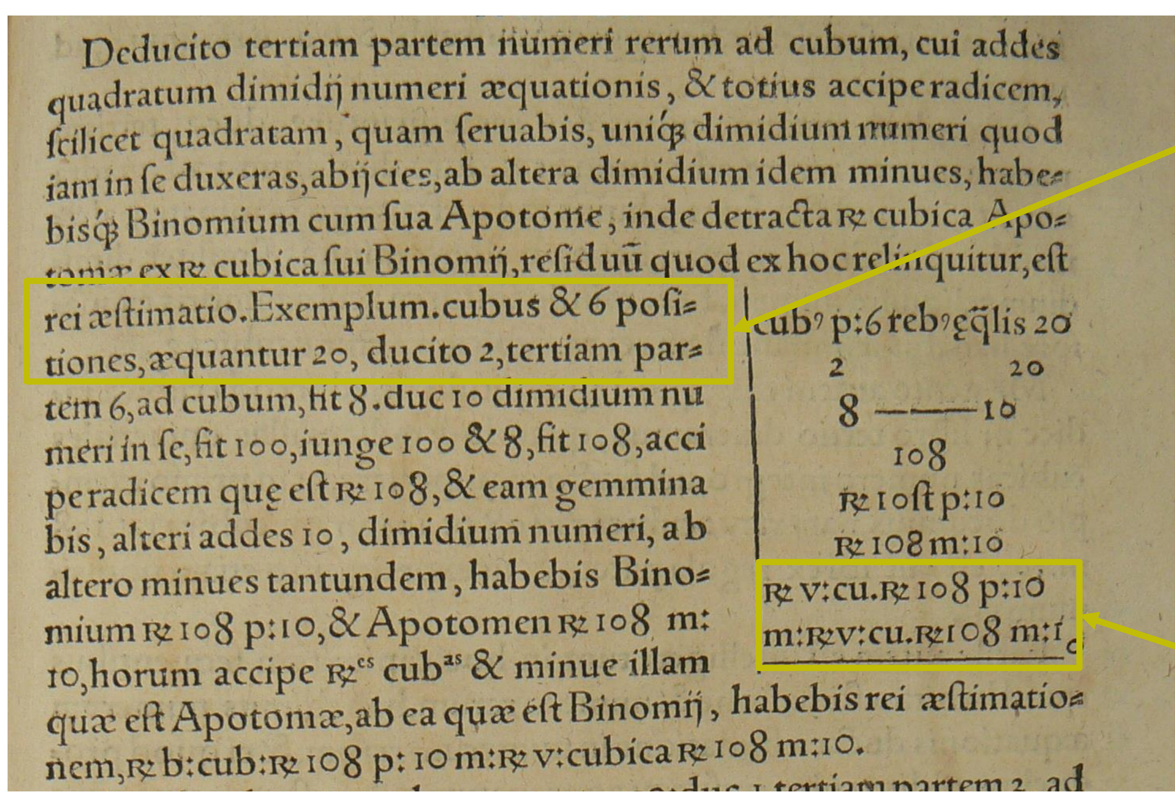
JEROME CARDAN (1501-1576)

Physician, inventor and Italian astrologer Jerome Cardan learned mathematics thanks to his mathematician father and then at the University of Pavia. Afterwards he pursued the study of medicine in Padua and taught mathematics at the University of Milan from 1534.

He is the creator of the device that bears his name, which was originally intended to keep ships' compasses horizontal.

His monumental work "*Artis sive regulis magnae algebraicis*" ("*Of the great Art or of the rules of algebra*", 1545) better known by the name "*Ars magna*" was inspired by the famous algebra treatise of Al Khwarizmi. The reading of the treatise is difficult because of his private algebraic symbolism.

He published the first method for solving equations of the 3^{rd} and 4^{th} degree, from the works of Tartaglia.



Deducito tertiam partem numeri rerum ad cubum, cui addes quadratum dimidij numeri æquationis, & totius accipe radicem, scilicet quadratam, quam seruabis, unique dimidium numeri quod iam in se duxeras, abijcies, ab altera dimidium idem minues, habebisque Binomium cum sua Apotome, inde detracta R cubica Apotomæ ex R cubica sui Binomij, residuũ quod ex hoc relinquitur, est rei æstimatio. Exemplum. cubus & 6 positiones, æquantur 20, ducito 2, tertiam partem 6, ad cubum, fit 8. duc 10 dimidium numeri in se, fit 100, iunge 100 & 8, fit 108, accipe radicem quę est R 108, & eam gemminabis, alteri addes 10, dimidium numeri, ab altero minues tantundem, habebis Binomium R 108 p:10, & Apotomen R 108 m:10, horum accipe R^es cub^as & minue illam quæ est Apotomæ, ab ea quæ est Binomij, habebis rei æstimationem, R b:cub:R 108 p:10 m:R v:cubica R 108 m:10.

cub⁹ p:6 reb⁹ ęq̃lis 20
2 20
8 —— 10
108
R 108 p:10
R 108 m:10
R v:cu.R 108 p:10
m:R v:cu.R 108 m:10

Extract from Artis magnae sive de regulis algebraicis
Alcazar Library, Marseille

1 cube and 6 positions are equal to 20

Return to solve the equation
$x^3+6x=20$
After having raised 2, the third of 6, to the cube, we get 8
Multiply by 10, half of the number by itself, namely 100, join 100 & 8 to get 108
Take the root, which is $\sqrt{108}$ & use this precious result twice.
The first time add 10, half of the number :
$\sqrt{108}+10$
The other time you diminish it by as much
$\sqrt{108}-10$
And you will get the Binomial and the Apotonomial,
Of these take the cube root and remove that which is that of the Apotonomial, from that which is the Binomial.
$\sqrt[3]{\sqrt{108}+10}$ - $\sqrt[3]{\sqrt{108}-10}$
You will have the estimation of the thing.

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