EMERGENCE

ARABIC ALGEBRA

Algebra

From the seventh to the twelfth centuries, algebra was the branch of mathematics dealing with equations of first or second degree with the aim of reducing them to a canonical form and of solving them. The major innovation is the introduction of the concept of "equation"

Arabic Algebra

Arabic algebra is that of the mathematicians who wrote their works in Arabic: Persians, Jews, Berbers, ...
They were found in Baghdad, in Khwarizm (Aral Sea), Egypt, Persia, Syria, North Africa, in the Iberian peninsula, …

800

Muhammad Al-Khwarizmi (780 – 850)

Solution of second degree equations

No symbolism, no coefficients or negative solutions, the algorithmic proof is demonstrated geometrically.
Using three terms, al-Mal (good), al jidhr (root) and al-'adad (number), which according to modern notation corresponds to x^2, x and number, Al-Khwārizmī defines six canonical equations with positive coefficients, according to an order which takes into account the nature and number of the elements on both sides of the equation. He sets out the method of solution of these equations and then geometrically proves the existence of positive solutions alone.

Muhammad Al Khwarizmi belongs to a group of mathematicians and philosophers who worked in the heart of the House of Wisdom in Baghdad at the end of the 8th century.
The word "**algebra**" (al-jabr) comes from the title of his masterpiece "*al-Jabr wa'l muqabala*" ("الكتاب المختصر في حساب الجبر والمقابلة" (*reduction and balancing*).

The word "**algorithm**" is nothing but a distortion of his name, meaning "a native of Khwarism".

900

Abū Kāmil (850 - 930)

Using irrational coefficients and roots.

Nicknamed the Egyptian calculator, his most important book is the Kitab al-Kamil fī l-Jabr (The Complete Book of algebra). He studied systems of several equations with several unknowns. His work inspired Fibonacci.

Abū Bakr-al-Karaji (953 - 1030)

Emergence of formal calculation

1000

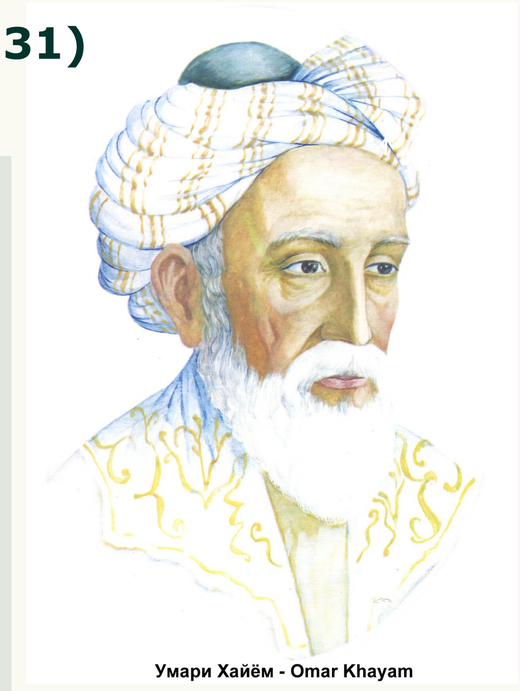
Applying calculations involving the knowns to the unknowns in order to establish a theory of formal calculation

Arabic algebra begins to free itself from geometry. Al-Karaji is aware that this break necessitates the founding of a new discipline on a basis permitting the definition of new objects (variables and abstract operations) and the justification of calculations independent of the axiomatic foundations of geometry.

His major treatises are *Al Fakhrī fi'l-jabr wa'l muqabala*, *Al-Badi fi'l hisab* et *Al-Kafi fi'l-hisab*. He uses Al Khwarizmi's theory of equations of the second degree, algebra in the tradition of Diophantus and that of Abu Kāmil. The decimal place value system and the study of the algebra of polynomials are found there. He laid down the rules for the four operations (+, -, x, :) by analogy with calculations on numbers. He introduces "recurrence" relations to demonstrate the formula for the sum of the first *n* cubes.

Omar al-Khayyām (1048 – 1131)

the poet and the cubic equations

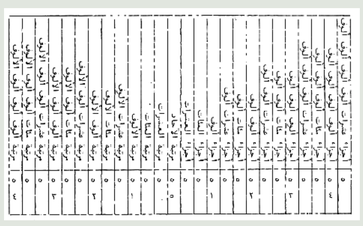

Умари Хайём - Omar Khayam

The Algebra of equations as a science in itself, quite distinct from the science of calculation and of geometry.
Still without symbolism, it operated a classification based on the degree and the number of terms. Like his predecessors, it does not envisage negative coefficients. He obtains twenty-five canonical equations. Eleven of these equations reduce to canonical forms of the second degree. That leaves fourteen, for which Omar Khayyam analyzes conditions for the existence of solutions (i.e. for real and positive roots) and establishes their existence.

1100

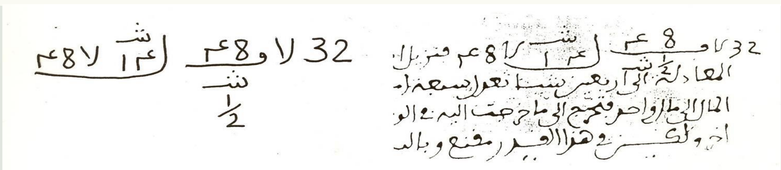
Al-Samaw'al (1130 - 1180)

Tables as objects of calculation
In his treatise al-Bahir fi'l-Jabr (The Brilliant in Algebra), he continues the work of Al Karaji and develops operational techniques on the polynomials, that he presents in the form of tables of coefficients. He uses negative exponents. He also uses reasoning by "recurrence".


Polynomials of al-Samaw'al from *wikimedia*

Symbolic Writing

The use of algebraic literal symbolism dates from the twelfth century; it began in Maghreb and El-Andalus


Anonymous manuscript containing an equation written with the aid of algebraic symbols used in the Maghreb between the 12th and 14th centuries. Iconography CNRS

1200

Sharaf al-Dīn al-Tūsī (died in 1213)

Classification of cubic equations according to their number of solutions, use of an auxiliary equation to establish the existence of the solutions.

This auxiliary equation corresponds exactly to the second order polynomial that would be obtained today by differentiating the cubic equation and setting this derivative to zero. But in the work which has come down to us, nothing is said about the steps that led to this auxiliary equation

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