

# **DIOPHANTE OF ALEXANDRIA** FATHER OF ALGEBRA?

Greek mathematician, born in Syria, who probably lived around the third century AD in Alexandria. He is the author of three books including the best known Arithmetica which deals with problem solving.



# THE ARITHMETICA

Work composed of thirteen books (according to Diophantus); until the discovery in 1968 in Iran of four new books in Arabic, only six were known; they came from a Greek manuscript discovered in 1464 by Regiomontanus in Venice. This book, little known by the Greeks has not been translated before the 10<sup>th</sup> century by Arab scholars, and then spread by them, some seeing it as an algebra book. Later on, 16<sup>th</sup> and 17<sup>th</sup> centuries' Western mathematicians took it over, especially **Pierre de Fermat**.

DIOPHAN-TI ALEXANDRINI ARITHMETICORVM LIBRI SEX, ET DE NVMERIS MVLTANGVLIS LIBER VNVS. CVM COMMENTARIIS C.G. BACHETI V.C. & observationibus D.P. de FERMAT Senatoris Tolosani. Acceffit Doctrinæ Analyticæ inuentum nouum, collectum ex varijs eiusdem D. de FERMAT Epistolis. ALS SEPTEM DIE TOLOSE Excudebat BERNARDVS BOSC, è Regione Collegij Societatis Iefu. M. DC. LXX.

EMERGENCE

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Arithmetica, page 1, Vatican Library

#### **Collection of 189 arithmetic or geometric**

**problems** not related to everyday life situations, reducible (in modern language) to first and second degree equations for which he searches for whole or positive fractional solutions, unlike Archimedes of Syracuse (- 287; -212) or Heron of Alexandria (1<sup>st</sup> century) who admitted irrational solutions.

Cover page of the 1670 edition of Arithmetica

THE USE OF SYMBOLS The symbols ("abbreviated designations") of **Diophantus:**  $\Delta^{y}$  named square (a<sup>2</sup>)  $K^{\gamma}$  named cube ( $a^{3}$ )  $\Delta^{\gamma}\Delta$  named square-square (a<sup>4</sup>)  $\Delta K^{\gamma}$  named square-cube (a<sup>5</sup>)  $K^{\gamma}K$  named Cubo-cube ( $a^{6}$ ). the arithme,  $\zeta$  noted, is an "unknown number", the forerunner of the unknown?

WHERE WE SEE FOR THE FIRST TIME

• He uses the Greek alphabetical notation for numbers:  $\alpha = 1, \beta = 2, \gamma = 3, ...$ He first writes the monomials with

sadere to, 1er igitur 2, adici. Amaile & a" H' 1. Ing sith ce" & And tis mitatibus to. zquatur 4 N. + 4. & S. s. jinne, i derbus u' 3". toui ut itiefit : N. 3. Erit ergo minor 3. maior 5. & our 4" 5. 6 de suifer 4" i. i meier es farisfaciunt qualitioni. wither

allum numerorum 2. minor autem c' bic, i des unfor four e' bie u' f. Mi 1 N. atque ideo maior 1 N. + 2. Oportet on der derbuie & producte f rendrationalisatitaque 4 N. + 4. triplos elle ad 2. & ad- Dur B. & Tri vargiegen pl i. rait der

IN QUAESTIONEM VIL

"ONDITIONIS appolitz cadem rattio eft que & appolitz precedenti quellioni, nil enim Cassad requirit qu'an ve quadratus internalli numerorum fit minor internallo quadratorum, & Cassaes iidem hic crian locum habebunt, ve manifefham eff.

#### QVESTIO VIIL

PROPOSITION quadratos anderetor fir ve 26. dividatur in duos quadratos, Ponatur dasses en d'is respersioner, and reraiges i 16. diuidatur in duos quadratos. Ponztur primus i Q.Oportetigitur 16 – 1 Q. aqua-les elle quadrato. Fingo quadratum à nu-meris quotquot libuerit, cum defectu tor vnitatum quod continet latus ipfius 16. efto à 2 N. – 4. ipie igitur quadratus erit,  $4 Q_{+} + 16. - 16$  N. hac aquabuntur vni-taubus 16 – 1 Q. Communis adiiciatur vtrimque defectus, & à fimilious auferan-tur fimilia, fient ș Q. aquales 16 N. & fie 1 N.  $\stackrel{n}{+}$  Erit igitur alter quadratorum  $\stackrel{n}{+}$ , alter verò  $\stackrel{n}{+}$  & vtriufque fumma eft  $\stackrel{n}{+}$  feu 16. & vterque quadratus eft. 16. & sterque quadratus eft. einermierTur. C e d'és sumbires main venescommenta, fou perietas of and for inderpor mentanto.

#### OBSERVATIO DOMINI PETRI DE FERMAT.

(Vonm antem in duos enbes , ant quadratoquadratum in duos quadratoquadratos Oc gener diter nullam in infnitum vitra quadratum poteffatem in duos einfdem nominis fer eft dinidere enins rei demonstrationem mirabilem fane detexi, Hanc marginis exignitas non caperer.

QVÆSTIO IX.

R dividere in duos quadratos. Pona- E2T O d'a main reir no mpayano detur rurfus primi latus i N. alterius verd ini warou maden e' inte, i ? m iripa quot cunque numerorum cum defectu tot vnitatum, quot conflat latus dividendi. Efto itaque 2 N. – 4. erunt quadrati, hie cuidem 1 Q. ille veriv 4 Q. + 16. - 16 N. Carterum volo verumque finul aquari vnitatibus 16. Iginur 5 Q. + 16. - 16 N. aquatur vnitatibus 16. & fit 1 N. Ferit  $\mu^2 \vec{e}$ . xel pinras é deslade si musifur. Hill

Page 61 of the 1670 edition of the Arithmetica of Diophantus. It contains the famous notes of Fermat on is great theorem.

- positive coefficients, then a separator M followed by the constant.
- He then places the separator  $\Lambda$  and the sequence of monomials with negative coefficients.

| How to write polynomials with these symbols?  |                           |                             |
|---|---------------------------|-----------------------------|
| Δ <sup><i>y</i></sup> γ   | matches                   | 3 <i>x</i> <sup>2</sup>     |
| ΔΚ <sup><i>y</i></sup> δΜη  |                           | 4 <i>x</i> <sup>5</sup> + 8 |
| ΔΚ <sup>γ</sup> δΚ <sup>γ</sup> εΔ <sup>γ</sup> κζλγΜη  | $4x^5 + 5x^3 + 20x^3$     | $x^{2} + 33x + 8$           |
| <b>Κ<sup>γ</sup>βΜη <u></u></b> <u></u> | 2 <i>x</i> <sup>3</sup> – | $x^2 - 2x + 8$              |

### **EXAMPLES OF PROBLEMS**

### **Problem 5 of book V:**

Find two numbers, one a cube and the other a square, such as if the cube of the cube is multiplied by two given numbers and added to

## AND NOW WHAT IS LEFT?

- **DIOPHANTINE EQUATIONS :**
- Equations with **integer coefficients** for which we try whole or rational solutions.
- **DECOMPOSITION OF A NUMBER INTO SUM OF TWO SQUARES:** He writes, without proving it:

each of these products the square of the square, the result is in each case a square. In modern notation:  $(x^2)^2 + a(y^3)^3 = u^2 et (x^2)^2 + b(y^3)^3 = v^2$ 

**Problem 16 of book I**,

Find three numbers which, taken in pairs, form the proposed numbers. In modern notation: x+y=a, y+z=b, z+x=c

«The whole in the form 4n + 1 can all be decomposed into two squares.».

# RIDDLE HOW LONG DID DIOPHANTUS LIVE?

#### Epitaphe



«Here lies Diophantus, the wonder behold. Through art algebraic, the stone tells how old: God gave him his boyhood one-sixth of his life One twelfth more as youth while whiskers grew rife; And then yet one-seventh ere marriage begun In five years there came a bouncing new son Alas, the dear child of master and sage after attaining half the measure of his father's life chill fate took him. After consoling his fate by the science of numbers for four years, he ended his life.»

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