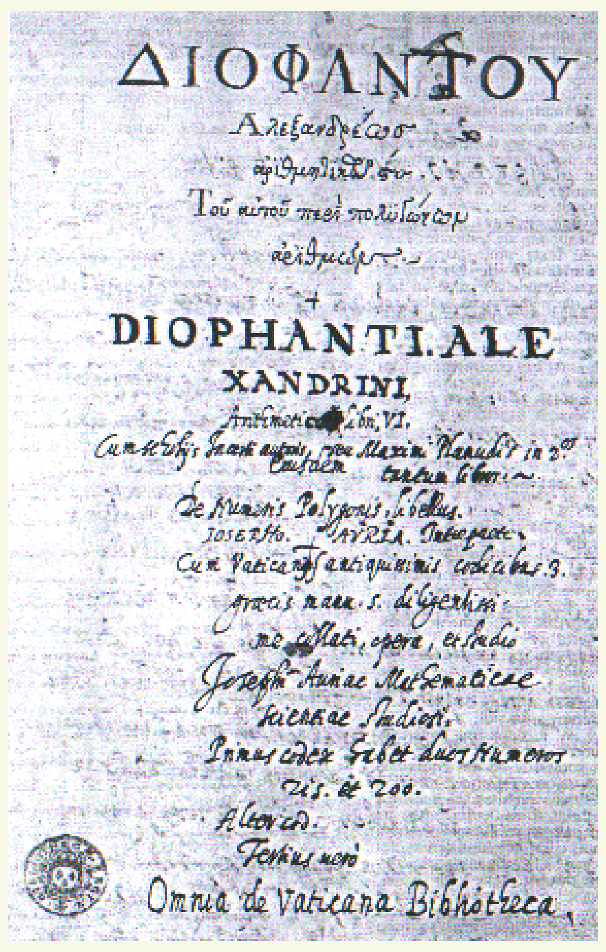
DIOPHANTE OF ALEXANDRIA

FATHER OF ALGEBRA ?

EMERGENCE

Greek mathematician, born in Syria, who probably lived around the third century AD in Alexandria. He is the author of three books including the best known Arithmetica which deals with problem solving.

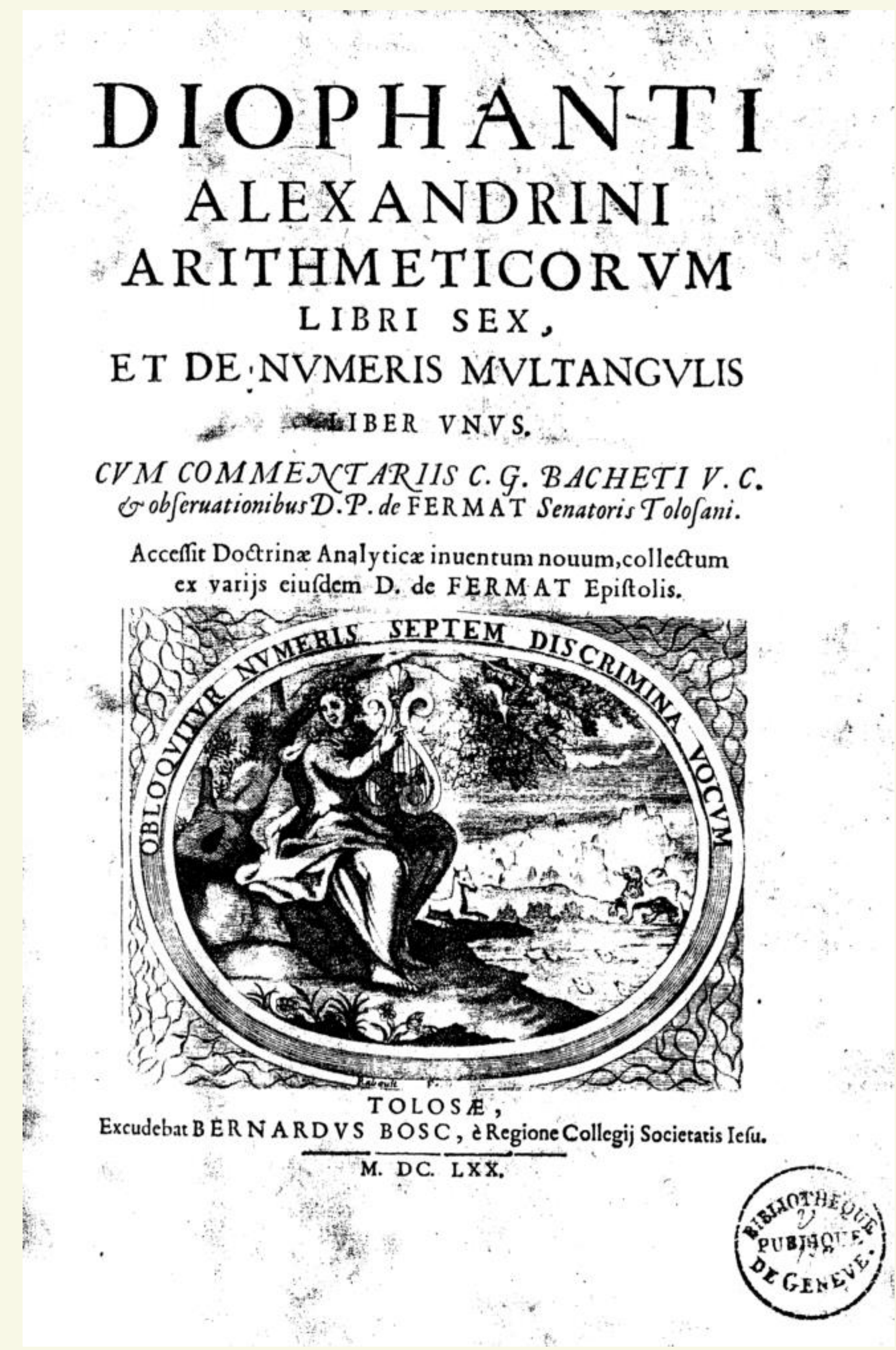


Arithmetica, page 1,
Vatican Library

The Arithmetica

Work composed of thirteen books (according to Diophantus); until the discovery in 1968 in Iran of four new books in Arabic, only six were known; they came from a Greek manuscript discovered in 1464 by Regiomontanus in Venice. This book, little known by the Greeks **has not been translated before the 10th century by Arab scholars**, and then spread by them, some seeing it as an algebra book. Later on, 16th and 17th centuries' Western mathematicians took it over, especially **Pierre de Fermat**.

Collection of 189 arithmetic or geometric problems not related to everyday life situations, reducible (in modern language) to first and second degree equations for **which he searches for whole or positive fractional solutions**, unlike Archimedes of Syracuse (- 287; -212) or Heron of Alexandria (1st century) who admitted irrational solutions.


DIOPHANTI
ALEXANDRINI
ARITHMETICORVM
LIBRI SEX,
ET DE NVMERIS MVLTANGVLIS
LIBER VNVS.
CVM COMMENTARIIS C. G. BACHETI V. C.
& obseruationibus D. P. de FERMAT Senatoris Tolosani.
Accessit Doctrinæ Analyticæ inuentum nouum, collectum ex varijs eiusdem D. de FERMAT Epistolis.

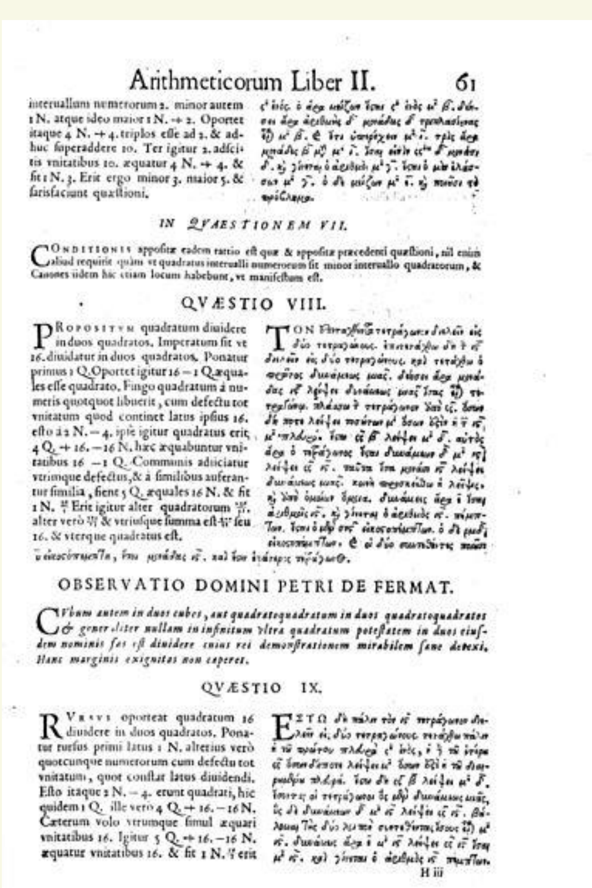
TOLOSÆ,
Excudebat BERNARDVS BOSC, è Regione Collegij Societatis Iesu.
M. DC. LXX.

Cover page of the 1670 edition of Arithmetica

Where we see for the first time the use of symbols

The *symbols* ("abbreviated designations") of Diophantus:
Δ^{γ} named square (a^2)
K^{γ} named cube (a^3)
$\Delta^{\gamma}\Delta$ named square-square (a^4)
ΔK^{γ} named square-cube (a^5)
$K^{\gamma}K$ named Cubo-cube (a^6).
the arithme, ζ noted, is an "unknown number", the forerunner of the unknown?

- He uses the Greek alphabetical notation for numbers:
  $\alpha = 1$, $\beta = 2$, $\gamma = 3$, …
- He first writes the monomials with positive coefficients, then a separator M followed by the constant.
- He then places the separator ⋔ and the sequence of monomials with negative coefficients.



Page 61 of the 1670 edition of the Arithmetica of Diophantus. It contains the famous notes of Fermat on is great theorem.

How to write polynomials with these symbols?

$\Delta^{\gamma}\gamma$	matches	$3x^2$
$\Delta K^{\gamma}\delta M\eta$		$4x^5 + 8$
$\Delta K^{\gamma}\delta K^{\gamma}\varepsilon\Delta^{\gamma}\kappa\zeta\lambda\gamma M\eta$		$4x^5 + 5x^3 + 20x^2 + 33x + 8$
$K^{\gamma}\beta M\eta$ ⋔ $\Delta^{\gamma}\alpha\zeta\beta$		$2x^3 - x^2 - 2x + 8$

Examples of problems

Problem 5 of book V:
Find two numbers, one a cube and the other a square, such as if the cube of the cube is multiplied by two given numbers and added to each of these products the square of the square, the result is in each case a square.
In modern notation:
$(x^2)^2 + a(y^3)^3 = u^2$ et $(x^2)^2 + b(y^3)^3 = v^2$

Problem 16 of book I,
Find three numbers which, taken in pairs, form the proposed numbers.
In modern notation:
x+y=a , y+z=b , z+x=c

And now what is left?

- **Diophantine Equations :**
  Equations with **integer coefficients** for which we try **whole or rational solutions**.
- **Decomposition of a number into sum of two squares:**
  He writes, without proving it:
  «The whole in the form 4n + 1 can all be decomposed into two squares.».

Riddle
How long did Diophantus live?

Epitaphe

«Here lies Diophantus, the wonder behold.
Through art algebraic, the stone tells how old:
God gave him his boyhood one-sixth of his life
One twelfth more as youth while whiskers grew rife;
And then yet one-seventh ere marriage begun
In five years there came a bouncing new son
Alas, the dear child of master and sage after attaining half the measure of his father's life chill fate took him.
After consoling his fate by the science of numbers for four years, he ended his life.»

IREM Aix-Marseille
http://www.irem.univ-mrs.fr/expo2013