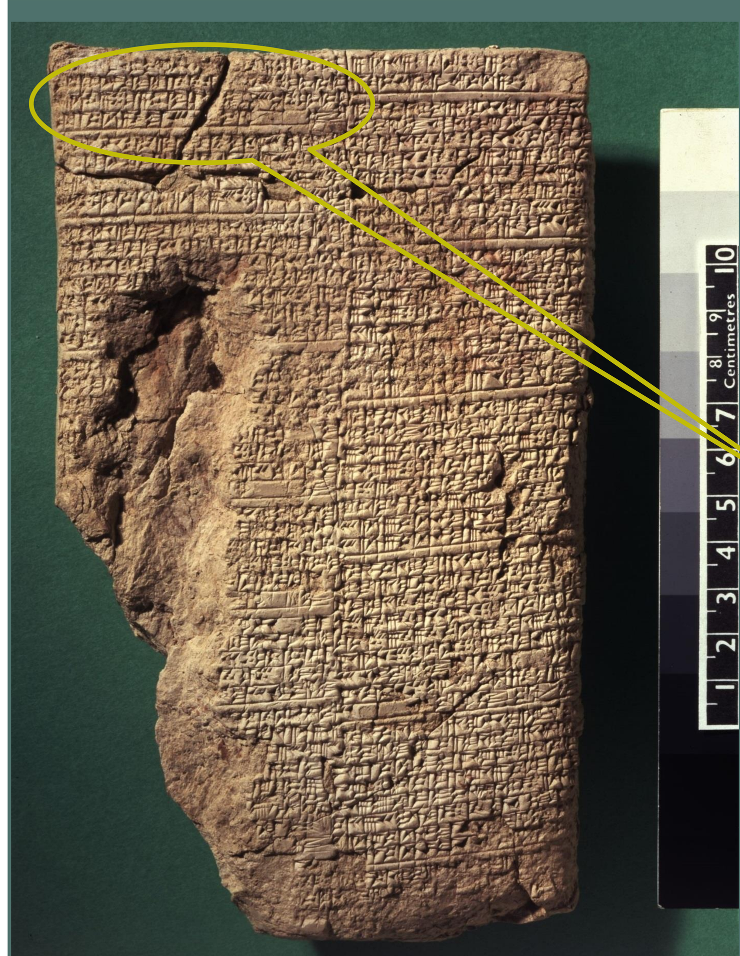
EMERGENCE

ALGEBRA ....
.... BEFORE THE LETTER

Today when we think of algebra, we immediately associate it with a symbolism (a recourse to unknown variables named by letters). But algebra, as a set of ways to solve problems, is much earlier. The following passages describe, through examples, a great variety of recipes for solving a variety of problems related to everyday life (shares, taxes, topography ....)

BABYLONIANS



Tablet 13901 of the British Museum (1850 BCE)

This is one of the oldest known mathematical texts. It lists 24 model solutions to equations constituting a veritable "manual of calculations of the second degree."

Example :

I have added the surface and (the side of) my square: 45'

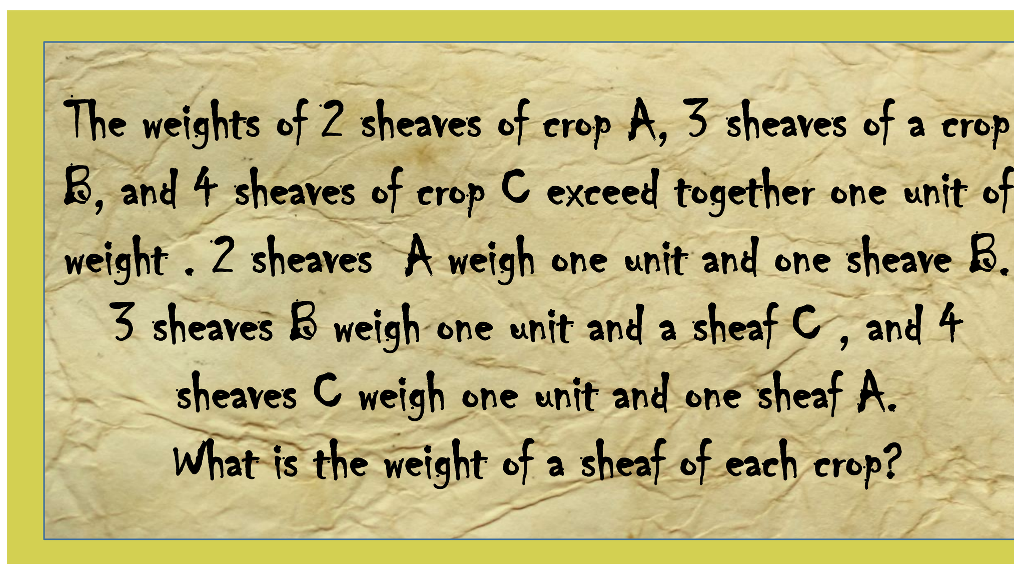
Model-Solution

- ✓ **You will write down 1 unit.**
- ✓ **You will divide up 1 into two: 30'.**
- ✓ **You will cross 30' and 30': 15'.**
- ✓ **You will add 15' to 45': 1'.**
- ✓ **This is the square of 1.**
- ✓ **You will subtract 30', that you have crossed, from 1: 30'**
- ✓ **the side of the square is 30'**

Today we would solve the equation: $x^2 + x = \frac{3}{4}$

Attention!
The Babylonians used base 60
45' is thus ¾

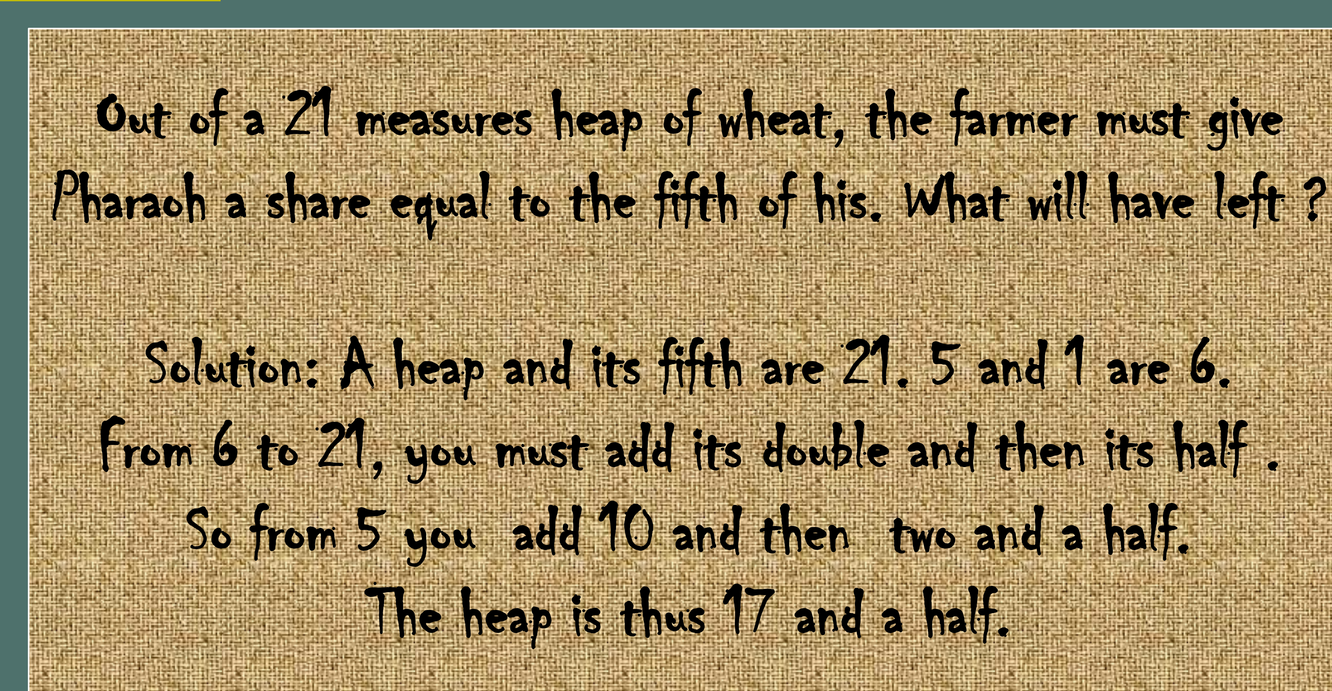
CHINESE



The weights of 2 sheaves of crop A, 3 sheaves of a crop B, and 4 sheaves of crop C exceed together one unit of weight . 2 sheaves A weigh one unit and one sheave B. 3 sheaves B weigh one unit and a sheaf C , and 4 sheaves C weigh one unit and one sheaf A. What is the weight of a sheaf of each crop?

Problem drawn from "Jiuzhang suanshu" (The nine chapters on the art of calculation). (200 BCE)

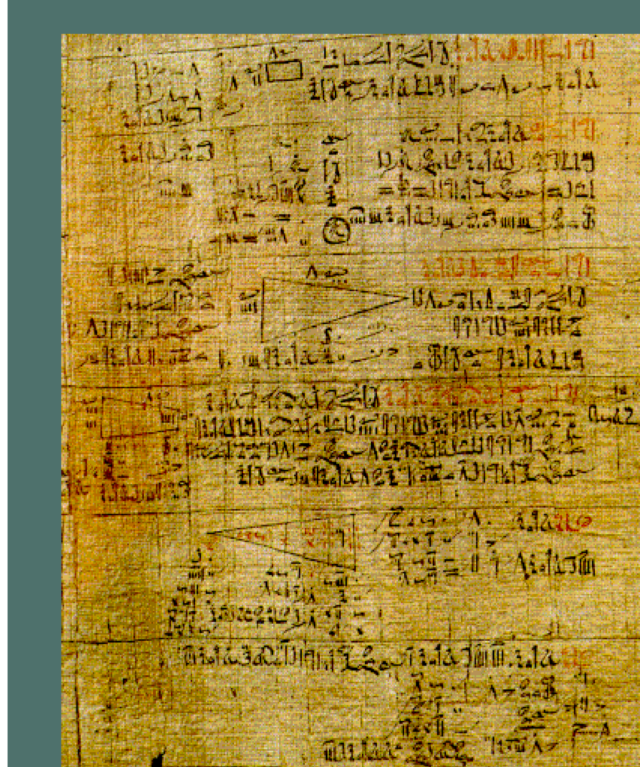
EGYPTIANS



Out of a 21 measures heap of wheat, the farmer must give Pharaoh a share equal to the fifth of his. What will have left ?

Solution: A heap and its fifth are 21. 5 and 1 are 6. From 6 to 21, you must add its double and then its half . So from 5 you add 10 and then two and a half. The heap is thus 17 and a half.

Excerpt from an Egyptian papyrus of the second millennium BCE



Rhind papyrus (1650 BCE)

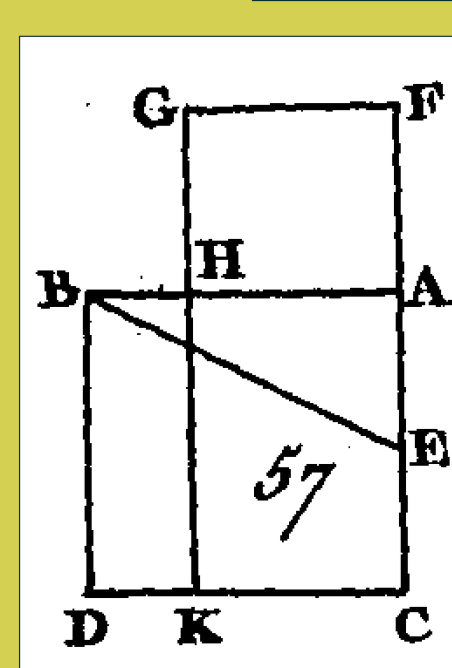
Today we would solve the equation :

$$x + \frac{x}{5} = 21$$

GRECS

« To cut a given straight line so that the rectangle contained by the whole and one of the segments equals the square on the remaining segment »

Euclide's Elements book II proposition XI



Today we would use notation $a = AB$ and $b = HB$ and solve the equation :

$$a(a - b) = b^2$$

See Euclid's solution in "to find out more"

Cylinder of Cyrus (British Museum) (5th century BCE)

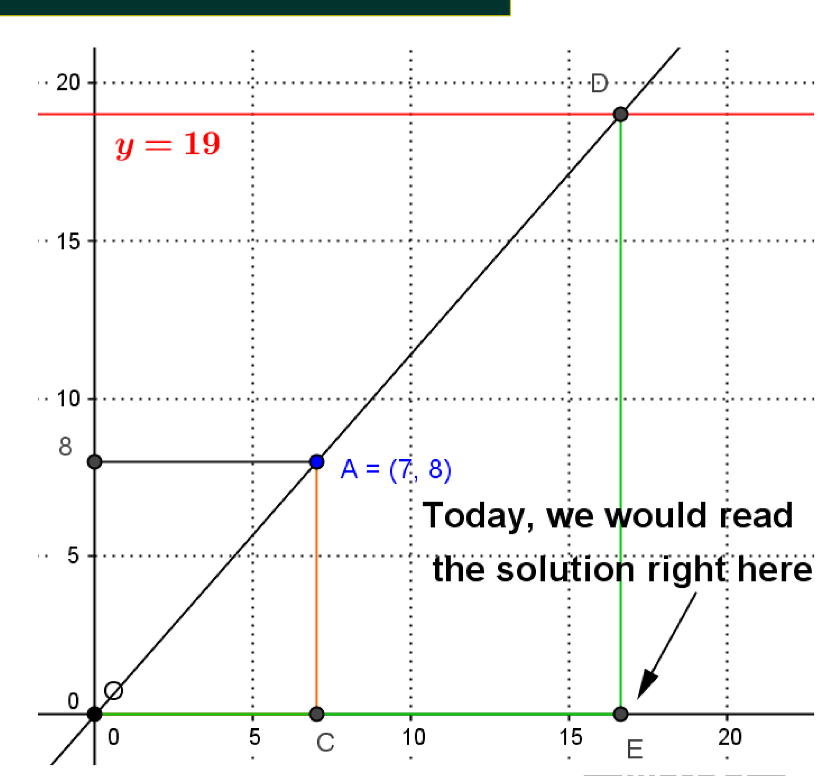
One method comes up very frequently in all these sources and seems to have been universally used for linear problems : The method of « false position »

Here for example, is problem No. 24 of the Rhind papyrus :
«An amount added to its seventh becomes 19. What is the amount ? »

Method :

- ✓ **We try with 7 (chosen as it is easy to divide by 7) :** $7 + \frac{7}{7}$ **which gives 8 while we want 19.**
- ✓ **By proportionality 7 is to the value sought what 8 is to 19, or** $\frac{7}{x} = \frac{8}{19}$
- ✓ **Thus the value sought is** $\frac{7 \times 19}{8} = \frac{133}{8}$

This method appears right up to the thirteenth century, notably in the "Liber Abaci" (1202) of Fibonacci and even in the fifteenth century. Using the method of double false position extends it to affine problems.



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