EMERGENCE

# GREECE, CRADLE OF PROOF

*L'École d'Athènes, Raphaël (détail) : Platon et Aristote , Musée du Vatican*

## ARISTOTLE (-384, -322) : the founding father of logic.

He was the first to seek for, and find, the principles of logic (the theory of syllogisms, the principle of non-contradiction, ...)

Aristotle, called the Stagirite, was a pupil of Plato at the Academy of Athens, the tutor of Alexander the Great and the founder of the School of Athens. He was interested in the arts and the sciences (biology, physics, ...).

In the "Organon", he introduced the concept of a proof based on "syllogisms", not for its application to mathematics, but as a precision instrument for use in dialogues and philosophical debates. For him, a correct proof was a proof that "wins" against all possible refutations.

The entirety of his work was translated into Latin from Arabic in the thirteenth century and was widely commented upon.

syllogism

*All athletes are human*
*All humans are mortal.*
*Therefore all athletes are mortal*

## THE STOïCS

(3rd century B.C.E.)
**They proposed rules of reasoning** for the logical connectives between propositions

## And before?

### THALES of Miletus

(-625,-546)
**He renounced gods and magical forces** in explaining the world order. He favoured observation and demonstration. That of the height of a pyramid would be a first in geometry.

### PYTHAGORAS

(-582,-500)
For him, "**Everything is number**". He searched for integers with specific properties to found the theory of numbers. He became interested in the concept of number, triangles and other mathematical figures and in the abstract idea of proof.

### PLATO

(-427,-347)
**brought out the abstract nature of mathematics.**
In one proof, he rejected recourse to experience. He insisted on precise definitions and clear hypotheses.

## And after ?

### EUCLID

(-323, -265)
In The Elements, a synthesis of the mathematics of his time, he sets out **axioms** and **postulates**. Then he **proves each property from those previously proven**.
The rules of logic he uses are mostly implicit, those of Aristotle and the Stoics not being sufficiently rich.

## What a proof?

**Aristotle Definition (*Topics Book I,1, 100a 25-27*)**

*« A syllogism is a form of argument in which, certain things being supposed, something distinct from those that were supposed necessarily follows, by virtue of what has been set out. It is a proof (apodeixis) is when the starting points of a syllogism are true and primary assertions, or at least assertions such that the knowledge we have of them arises through the intermediary of certain true and primary assertions; (...) »*

**Current Definition**

*A proof is a chain whose last link is the proposition to be demonstrated. The first links are the givens (axioms and hypotheses). We move from one link to another by applying previously accepted rules.*

*Organon, Aristotle, parchment Paris_end 13th, BNF*

## Reasoning known and practiced by the Greeks

### Reasoning by contradiction

Reasoning consisting in showing the impossibility of the proposition being false in order to conclude from this that it is true.
***Euclid uses it*** *to show that, in current terminology, √2 is irrational*

### Dialectic

Reasoning that sets up a thesis and its antithesis in parallel, the latter being defended within a dialogue between two interlocutors.
***Aristotle used it to*** *validate the law (the 'principle') of non-contradiction and to justify that the Earth is round by refuting that it could be square, triangular.....*

### Method of Exhaustion

This method of proof has been used over many centuries to calculate areas, volumes ... it consists in proving that desired magnitude can neither be greater than nor less than a certain value. It is the ancestor of the "method of indivisibles" and the "integral calculus" (17th century).
***Archimedes used it to*** *calculate the area under the arc of a parabola, to approximate π, and to work out the quadrature of the parabola.*
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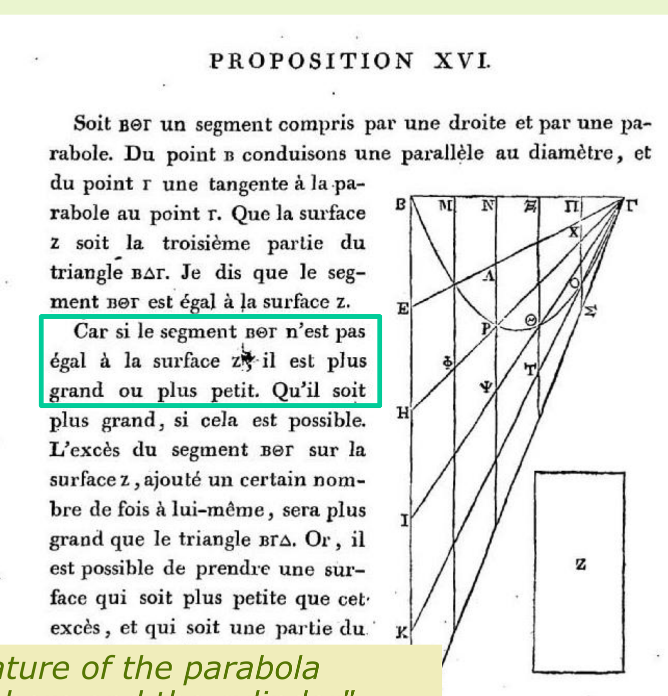

PROPOSITION XVI.

Soit ΒΘΓ un segment compris par une droite et par une parabole. Du point Β conduisons une parallèle au diamètre, et du point Γ une tangente à la parabole au point Γ. Que la surface Ζ soit la troisième partie du triangle ΒΔΓ. Je dis que le segment ΒΘΓ est égal à la surface Ζ.

Car si le segment ΒΘΓ n'est pas égal à la surface Ζ, il est plus grand ou plus petit. Qu'il soit plus grand, si cela est possible. L'excès du segment ΒΘΓ sur la surface Ζ, ajouté un certain nombre de fois à lui-même, sera plus grand que le triangle ΒΓΔ. Or, il est possible de prendre une surface qui soit plus petite que cet excès, et qui soit une partie du



*Quadrature of the parabola*
*"On the sphere and the cylinder", Archimedes.*
*trans. F. Peyrard, Paris 1807,*
*Source Gallica.bnf.fr*

## And if some of the rules and axioms used by Euclid are suppressed?

**In geometry: without the parallel postulate of Euclid**

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**Non Euclidean geometry**

Application : Maritime and aerial navigation

**In limiting ourselves to one, and only one use of hypotheses, in a proof.**

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**Linear Logic**

Application : Data management

**Without the law of the excluded middle (validity of a proposition or its contrary).**

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**Intuitionistic logic**

Application : Computing


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