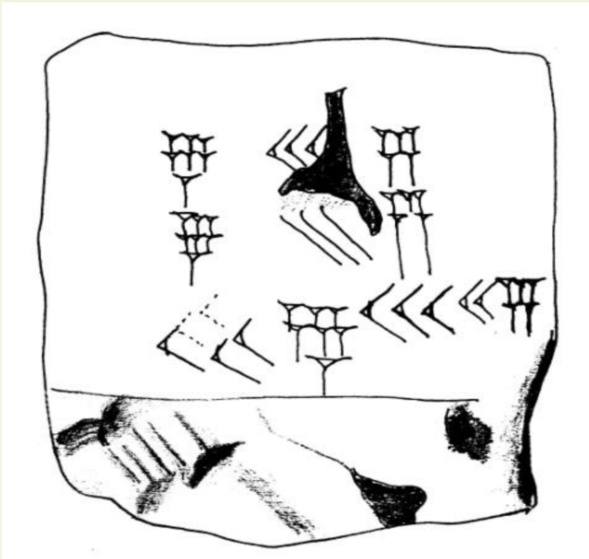
MEDITERRANEAN
RECKONING
MULTIPLICATIONS

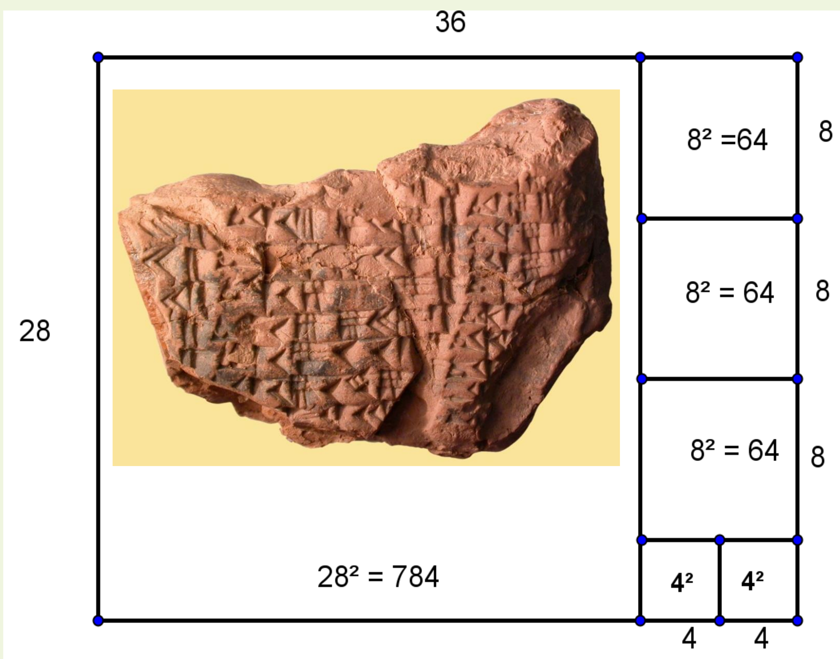
In Mesopotamia: We calculate in base 60; we evaluate surfaces



Ni 1024 a schoolboy tablet Nippur

Calculating a product

A number was considered as the length of a segment and **a product as the area of a rectangle.** To calculate a product, cut a rectangle into squares.



36 X 28 = 28²+ 8² +8² + 8² + 4² + 4² = **1008**

Every integer is decomposable into a sum of squares

Calculate the square of 455

455** = 7x 60 + 35 = **7.35 in base 60

The multiplication posed by the scholar:

7. 35
7. 35
57. 30. 25

57x 3600 + 30 x60 + 25 = 207 025 = **455²**

The pupil didn't get it wrong !

In ancient Egypt: We multiply and divide by 2

Example : 28 x 36

	36	28	
x2	72	14	: 2
x2	**144**	7	: 2
x2	**288**	3	: 2
x2	**576**	1	: 2

A first algorithm
(often called Russian multiplication)

The looked-after product is equal to the sum of the numbers on the left side which are opposite odd numbers

28 x 36 = 144 + 288 + 576 = **1008**

Example: calculation of : 28 x 36

28 = 4 + 8 + 16

	36	1	
x2	72	2	x 2
x2	**144**	4	x 2
x2	**288**	8	x 2
x2	**576**	16	x 2

A second algorithm

Any integer can be expressed as a sum of powers of 2

The looked-after product is equal to the sum of the numbers on the left side which are opposite the numbers appearing in the decomposition of 28 :

28 = 16 + 8 + 4

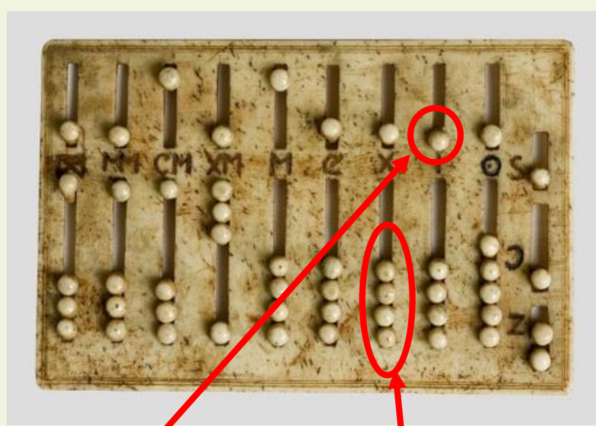
28 x 36 = 144 + 288 + 576 = **1008**

By the Greeks and Romans, up to « calculi »

To perform calculations the Greeks and Romans used abacuses ("tables for counting") and tokens, often little stones.

The value of a token depends on:

- Its **position on one of the vertical lines** of the abacus (units, tens, hundreds, ...)
- **It is worth a unit if located below the horizontal separation and five units if it is located above.**



5 tens 4 hundreds

The Latin word for "Stones" was "calculi" Hence our word "calculation"

Georg Reisch, 1508 Margarita Philosophica

The use of these abacuses persisted in Europe until the Renaissance and even until the French Revolution.

But algebraists (represented here by Boethius on the left) eventually took over from abacistes (represented here by Pythagoras on the right) getting the favor of the arithmetic muse (at the center)

The Nasir ad Dīn al Tusi's algorithm (1201 – 1274)
Or how to avoid carries

Example :

To multiply 352 by 4, we proceed as follows:

352
4
1208
20
1408

Step 1 :
Multiply 4 by 2 (= 08) then by 3 (= 12), yielding 1208.

Step 2 :
Multiply 4 by 5 (= 20), write 20 on the next line, shifting a row.

Final step: Add 1 208 and 20

16th century copy of the manuscript of the Arabic mathematician al-Qualasadi. Notice that he was still using the Indi digits (source : BNF)

The "per Gelosia" multiplication, Italian Renaissance

Used by Arabic mathematicians as far as the 8th century, **Fibonacci (Leonardo of Pisa)** introduced this algorithm in Europe in 1201, in his book **"Liber Abaci".** It was used until the 17th century.

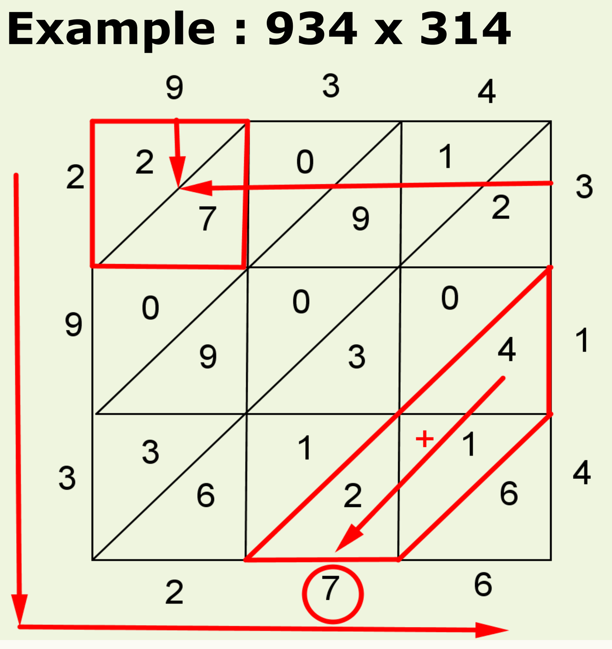
The name "per gelosia" refers to venetian blinds (called "jalousie" in French) as it is done in a grid array.

The algorithm:

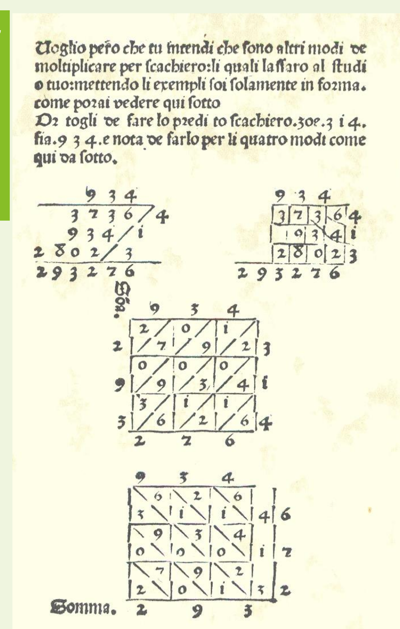
a. Write the numbers to be multiplied on the sides of the board, from left to right and from bottom to top.
b. Calculate all the partial products
c. Add the numbers in the "diagonal stripes" starting from the bottom right
d. Collect the result's digits starting from the top left

"But I want you to understand that there are other ways to multiply by checkerboard, which I leave you look for. I leave you only see the following examples. "

Example : 934 x 314



934 X 314 = 293 276



Anonymous manuscript of 1478, written in Italian, found in Treviso.

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